# Chapter 2

# Mathematics Achievement in the Cognitive Domains at the Fourth and Eighth Grades

This chapter of the report presents the TIMSS 2003 mathematics achievement results for each of the three cognitive domains. Following the presentation of the results, for each domain in turn – knowing, applying, and reasoning – there is an overview of performance across domains.

## Knowing Facts, Procedures, and Concepts

The first page of Exhibit 2.1 presents the distribution of students' mathematics achievement in the cognitive domain of knowing facts, procedures, and concepts for the 46 countries and four benchmarking entities that participated in TIMSS 2003 at the eighth grade, and the second page presents the distribution of student achievement for the 25 countries and three benchmarking entities that participated at the fourth grade. Countries are shown in decreasing order of average (mean) scale score, together with an indication of whether the country average is significantly higher or lower than the international average. To provide a basis of comparison for the performance of each country in each cognitive domain, the international average across countries for each domain was scaled to be 467, the same as the international average for mathematics overall. As explained in Chapter 1 (footnote 1), the benchmarking entities were not included in computing the international average. Also, as previously discussed in conjunction with Exhibit 1.1, the years of formal schooling and average age of the students in each country are shown to aid in interpretation of the achievement results. This information also is repeated in Exhibit 2.1 as well as in 2.3 and 2.5 for ease of reference.

Finally, as a reminder that not all countries are equally well equipped to meet the challenge of educating their young people, Exhibit 2.1 as well as Exhibits 2.3 and 2.5 include the value for each country on the Human Development Index provided by the United Nations Development Programme (see Human Development Report 2003). The index has a minimum value of 0 and a maximum of 1.0. Countries with high values on the index enjoy long life expectancy, high levels of school enrollment and adult literacy, and a good standard of living as measured by per capita GDP. For example, at the eighth grade, TIMSS countries with index values greater than 0.9 included Australia, Belgium (Flemish), England, Israel, Italy, Japan, New Zealand, Norway, The Netherlands, Scotland, Sweden, and the United States. For all three cognitive domains, all of these countries (except Norway in the knowing and applying domains) had average achievement above the international average. However, not all countries performing above the overall international average in the three cognitive domains had an index value as high as 0.9. Within each of the cognitive domains, the relationship between a country's index value and average student achievement was fairly similar.

As shown in Exhibit 2.1, in the knowing domain for the eighth grade, similar to overall mathematics performance, there was a wide range in performance between the highest- and lowest-performing countries, from 592 in the Republic of Korea to 232 in Ghana. Twenty-seven countries and the four benchmarking entities performed above the international average and 17 countries scored below the international average. Moldova and Cyprus performed about the same as the international average.

At the fourth grade, the difference was also large between the highest-performing country Singapore (626) and the lowest-performing country Tunisia (338). Thirteen countries and the three benchmarking entities performed above the international average and eight countries performed below the international average. The four countries performing about at the international average were Australia, Moldova, Cyprus, and New Zealand.

For both the eighth and fourth grades, Exhibit 2.1 illustrates the broad range of achievement both within and across the countries assessed. It provides a graphical representation of student performance within each country. The bar graph for each country shows the 5th, 25th, 75th, and 95th percentiles<sup>1</sup> as well as the 95% confidence for the mean. Each percentile point indicates the percentage of students below that point on the scale. For most TIMSS 2003 participants at the eighth grade, there was an enormous range within each country between the highest and lowest scores, often as much as 400 scale-score points. This range was as large or larger than the difference in mean achievement between the highest and lowest performing country. For the eighth grade knowing scale, the range for most students in the higher-achieving countries was from 400 to 700. In comparison, it tended to be between 300 and 600 for medium-performing countries and from 200 to 500 (or even lower) in the lower-performing countries.

Exhibit 2.2 shows how a country's average mathematics achievement in the knowing domain compares to achievement in the other participating countries. The results for the eighth grade are shown on the first two pages and for the fourth grade on the third page. The figure for each grade shows whether or not the differences in average achievement between pairs of countries are statistically significant. To read the table, select a country of interest from the first column and read across the row corresponding to that country. A circle with a triangle pointing up indicates significantly higher performance than the comparison country listed across the top; absence of a symbol indicates

<sup>1</sup> Tables of the percentiles values and standard deviations for all countries are presented in Appendix D.

#### **TIMSS2003**

#### Exhibit 2.1: Distribution of Mathematics Achievement for Knowing Cognitive Domain



Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
🕶 Korea, Rep. of	8	14.6	and a second	592 (2.1)	0.879
Singapore	8	14.3		591 (3.1)	ن 0.884
<sup>†</sup> Hong Kong, SAR	8	14.4		589 (3.3)	0.889
Chinese Taipei	8	14.2	and the second second second	585 (4.5)	- 5
Japan	8	14.4		564 (1.9)	0.932
Estonia	8	15.2		538 (2.7)	0.833 it
Belgium (Flemish)	8	14.1		537 (2.5)	0.937
Hungary	8	14.5		536 (3.1)	0.837
† Netherlands	8	14.3		520 (3.1)	0.938
Russian Federation	7 or 8	14.2	and a second	519 (3.4)	0.779 .5
Latvia	8	15.0		518 (2.8)	0.811
Slovak Republic	8	14.3	the second se	517 (3.3)	0.836
<sup>1</sup> Lithuania	8	14.9		511 (2.7)	0.824
<sup>‡</sup> United States	8	14.2		510 (2.8)	0.937
Malaysia	8	14.3		506 (3.9)	0.790
<sup>2</sup> Israel	8	14.0	and the second	501 (3.1)	0.905
Slovenia	7 or 8	13.8		499 (2.2)	0.881
Australia	8 or 9	13.9		497 (4.0)	0.939
<sup>1</sup> Serbia	8	14.9		495 (2.7)	-
Sweden	8	14.9		486 (2.1)	0.941
Bulgaria	8	14.9		486 (4.1)	0.795
Romania	8	15.0		485 (4.9)	0.773
New Zealand	8.5 - 9.5	14.1		485 (4.8)	0.917
Italy	8	13.9		484 (3.2)	0.916
<sup>†</sup> Scotland	9	13.7		481 (3.2)	0.930
Armenia	8	14.9		480 (2.9)	0.729
International Avg.	8	14.5		467 (0.5)	-
Moldova, Rep. of	8	14.9		466 (4.1)	0.700
Cyprus	8	13.8		466 (2.0)	0.891
Norway	/	13.8		450 (2.1)	0.944
Lebanon	8	14.6		447 (3.2)	0.752
<sup>2</sup> Macedonia, Rep. of	8	14.6		447 (3.8)	0.784
Jordan	8	13.9		428 (4.7)	0.743
' Indonesia	8	14.5		422 (4.3)	0.682
Egypt	8	14.4		411 (3.4)	0.648
Iran, Islamic Rep. of	8	14.4		405 (2.6)	0.719
Tupicio	0	14.1		401 (2.3) <b>•</b>	0.659
Palostinian Nat'l Auth	0	14.0		201 (2.7)	0.740
Philippines	0	14.1		388 (5.2)	0.751
	8	14.8		386 (3.2)	0.751
Chile	8	14.2		386 (3.2)	0.000
Botswana	8	15.1		372 (2.8)	0.614
Saudi Arabia	8	14.1		315 (4.6)	0.769
South Africa	8	15.1		261 (5.4)	0.684
Ghana	8	15.5		232 (5.9)	0.567
<sup>‡</sup> England	9	14.3		489 (4.0)	0.930
Benchmarking Participants	_			, -	
Basque Country, Spain	8	14.1		495 (2.2)	-
Indiana State, US	8	14.5		515 (4.6)	-
Ontario Province, Can.	8	13.8		513 (2.6)	-
Quebec Province, Can.	8	14.2		537 (2.7)	-
			0 100 200 300 400 500 600 700 800	D	

Country average significantly higher than international average

Country average significantly lower than international average

 $^{\ast}$   $\,$   $\,$  Represents years of schooling counting from the first year of ISCED Level 1.

- \*\* Taken from United Nations Development Programme's Human Development Report 2003,
- p. 237-240.
- t Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- \* Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- 2 National Defined Population covers less than 90% of National Desired Population (see Exhibit C.1).
- ➡ Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
- () Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
- A dash (-) indicates comparable data are not available.

75th 95th

Percentiles of Performance 25th 75th

95% Confidence Interval for Average (±2SE)

5th 25th

#### **TIMSS2003**

#### Exhibit 2.1: Distribution of Mathematics Achievement for Knowing Cognitive Domain

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Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
Singapore	4	10.3		626 (6.5)	0.884
<sup>†</sup> Hong Kong, SAR	4	10.2		574 (3.3)	0.889
Chinese Taipei	4	10.2		565 (2.2)	-
Japan	4	10.4		564 (2.1)	0.932
Belgium (Flemish)	4	10.0		558 (2.1)	0.937
<sup>†</sup> England	5	10.3	and the second	534 (4.5)	0.930
<sup>†</sup> Netherlands	4	10.2		530 (2.2)	0.938
<sup>†</sup> United States	4	10.2		528 (2.5)	0.937
<sup>1</sup> Lithuania	4	10.9	and a second	519 (2.7)	0.824
Hungary	4	10.5	the second se	517 (3.3)	0.837
Latvia	4	11.1		517 (2.9)	0.811
Italy	4	9.8	and a second	514 (3.9)	0.916
Russian Federation	3 or 4	10.6		513 (5.3)	0.779
<sup>†</sup> Australia	4 or 5	9.9		501 (3.8)	0.939
Moldova, Rep. of	4	11.0		500 (5.2)	0.700
Cyprus	4	9.9		500 (2.8)	0.891
International Avg.	4	10.3		495 (0.7)	-
New Zealand	4.5 - 5.5	10.0		493 (2.2)	0.917
<sup>†</sup> Scotland	5	9.7		484 (3.0) 💿	0.930
Slovenia	3 or 4	9.8		470 (2.6) 💿	0.881
ø Norway	4	9.8		448 (2.1) 💿	0.944
Armenia	4	10.9		447 (3.7) 💿	0.729
Iran, Islamic Rep. of	4	10.4		404 (4.0) 💿	0.719
Philippines	4	10.8		385 (6.9) 💿	0.751
Morocco	4	11.0		360 (4.4) 💿	0.606
Tunisia	4	10.4		338 (4.2) 💿	0.740
Benchmarking Participants					
Indiana State, US	4	10.5		544 (3.7)	-
Ontario Province, Can.	4	9.8		514 (4.4)	-
Quebec Province, Can.	4	10.1		504 (2.8)	-
			D         100         200         300         400         500         600         700         300           Percentiles of Performance         9         9         9         9         9         9         9         9         9         9         9         9         9         9         9         9         100         300         300         400         500         600         700         300         300         400         500         600         700         300         300         400         500         600         700         300         300         400         500         600         700         300         300         400         500         600         700         300 <th< td=""><td>300 Country average sign than international av</td><td>ificantly higher erage</td></th<>	300 Country average sign than international av	ificantly higher erage

95% Confidence Interval for Average (±2SE)

 Country average significantly lower than international average

\* Represents years of schooling counting from the first year of ISCED Level 1.

- \*\* Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
- Met guidelines for sample participation rates only after replacement schools were included (see t Exhibit C.2).
- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
- () Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates comparable data are not available.

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#### Multiple Comparisons of Average Mathematics Achievement for Knowing Exhibit 2.2: **Cognitive Domain**

Instructions: Read across the row for a achievement of the country in the row statistically significant difference betw	a count is sigr veen th	try to nifica ie av	o com antly rerag	npare lowe e acl	e per er tha nieve	form an th men	ance at of t of t	with the he ty	n the com wo c	cou paris ount	ntrie on co ries.	s list ount	ed al ry, si	ong gnifi	the t cantl	op o y hig	f the Jher t	char than	't. Th that	e syr of th	nbol ie co	s ind mpa	icate rison	whe cou	ether ntry,	the a or if	avera there	age e is n	0	
Countries	Korea, Rep. of	Singapore	Hong Kong, SAR	Chinese Taipei	Japan	Estonia	Belgium (Flemish)	Hungary	Netherlands	Russian Federation	Latvia	Slovak Republic	Lithuania	United States	Malaysia	Israel	Slovenia	Australia	Serbia	England	Sweden	Bulgaria	Romania	New Zealand	Italy	Scotland	Armenia	Moldova, Rep. of	Cyprus	Norway
Korea, Rep. of					٥	٥	٥	0	٥	٥	0	٥	٥	٥	0	٥	0	0	٥	0	٥	0	0	0	0	0	0	0	0	٥
Singapore					٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥
Hong Kong, SAR	_				0	0	0	0	0	0	٥	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Chinese Taipei	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Japan						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Belgium (Elemish)	•	•	•	•	•				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hungary	۲	۲	۲	۲	$\overline{\mathbf{v}}$				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Netherlands	۲	۲	۲	۲	۲	۲	۲	۲					0	0	0	٥	٥	0	٥	٥	٥	٥	0	0	٥	0	0	٥	0	٥
Russian Federation	۲	lacksquare	۲	۲	۲	lacksquare	lacksquare	۲						٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥
Latvia	۲	۲	۲	۲	۲	۲	۲	۲						0	٥	٥	٥	٥	٥	٥	٥	٥	0	0	٥	٥	0	٥	٥	٥
Slovak Republic						•	•		0						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Lithuania																0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
United States																0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Israel	•	•	•	•	•	•	•	•	•	•	•	•	$\overline{\mathbf{v}}$						•	0	0	0	0	0	0	0	0	0	0	0
Slovenia									۲		۲									0	0	0	0	0	0	0	0	0	0	0
Australia	۲	۲	۲	۲	$\overline{\mathbf{v}}$			۲		۲	۲	۲	۲							-	0				0	0	0	0	0	0
Serbia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲						٥	٥			0	0	0	٥	٥	٥
England	۲	lacksquare	۲	۲	۲	lacksquare	lacksquare	۲	lacksquare	۲	$\odot$	$\overline{\bullet}$	$\overline{\mathbf{v}}$	lacksquare	۲	$\odot$	۲											٥	٥	٥
Sweden	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲									٥	٥	٥
Bulgaria	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		۲						_			٥	٥	٥
Romania	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲				_							0	0	0
New Zealand																		0	0						_			0	0	0
Italy																												0	0	0
Armenia	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•									0	0	0
Moldova, Rep. of					•							•			•			•										•		0
Cyprus	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			0
Norway	۲	lacksquare	۲	۲	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	۲	lacksquare	$\overline{\bullet}$	$\overline{\mathbf{v}}$	lacksquare	۲	lacksquare	۲	lacksquare	lacksquare	۲	lacksquare	۲	lacksquare	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	lacksquare	۲	
Lebanon	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	
Macedonia, Rep. of	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	
Jordan	•	•					•	•																				•		
Indonesia																														
Egypt Iran Islamic Rep. of		•		•			•			•	•			•		•					•									
Bahrain	•					۲																				•				•
Tunisia	۲	$\overline{\bullet}$	۲	۲	$\overline{\mathbf{v}}$	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\mathbf{v}}$	۲	۲	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\bullet}$	۲	$\overline{\bullet}$	۲	$\overline{\mathbf{v}}$	$\overline{\bullet}$	۲	$\overline{\bullet}$	۲	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	۲	۲	$\overline{\bullet}$
Palestinian Nat'l Auth.	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Philippines	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	$\odot$	۲	۲	۲	۲	$\odot$	۲	۲	۲	۲	۲	۲	۲	۲	۲
Morocco	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Chile						•																								
Botswana																														
Saudi Arabia	•	•			•	•	•	•	•		•	•				•			•		•	•		•	•	•	•	•		•
South Airita Ghana		•	•	•		•	•	•	•	•		•	•	•	•	•	•	•		•		•		•	•	•	•	•	•	•
Benchmarking Participants					5		-	-											5											-
Basque Country, Spain	$\overline{\mathbf{v}}$		$\overline{\bullet}$		$\overline{\mathbf{v}}$				$\overline{\mathbf{v}}$			$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$		$\overline{\mathbf{v}}$						٥	0			0	0	0	0	0	٥
Indiana State, US	۲	۲	۲	۲	۲	۲	۲	۲								٥	0	0	٥	0	٥	0	0	0	٥	0	0	0	0	0
Ontario Province, Can.	۲	۲	۲	۲	۲	۲	۲	۲								٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥
Quebec Province, Can.	۲	۲	۲	۲	۲				0	٥	٥	0	0	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	0	0	0	0	٥	٥	٥

## Exhibit 2.2: Multiple Comparisons of Average Mathematics Achievement for Knowing Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Lebanon	Macedonia, Rep. of	Jordan	Indonesia	Egypt	Iran, Islamic Rep. of	Bahrain	Tunisia	Palestinian Nat'l Auth.	Philippines	Morocco	Chile	Botswana	Saudi Arabia	South Africa	Ghana		Basque Country, Spain	Indiana State, US	Ontario Province, Can.	Quebec Province, Can.	Countries
0	٥	0	٥	٥	0	٥	٥	0	٥	0	٥	٥	٥	٥	٥		0	٥	٥	0	Korea Ren of
0	٥	0	٥	0	0	٥	0	0	0	0	٥	٥	0	0	٥		0	0	0	0	Singapore
٥	٥	0	٥	٥	٥	٥	0	0	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥	0	Hong Kong SAR
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	Chinese Taipei
0	٥	0	٥	0	٥	٥	0	0	0	٥	٥	٥	0	0	٥		٥	0	0	0	Japan
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥		Estonia
٥	٥	0	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	0	0	٥		٥	0	0		Belgium (Flemish)
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥		Hungary
٥	٥	0	٥	0	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥		٥			۲	Netherlands
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥			۲	Russian Federation
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥			۲	Latvia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥			۲	Slovak Republic
٥	٥	0	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	0	٥	٥		٥			۲	Lithuania
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥			۲	United States
٥	٥	0	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥		٥			۲	Malaysia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	Israel
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	Slovenia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	Australia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	Serbia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	England
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	۲	Sweden
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	۲	Bulgaria
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	Romania
0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			۲	۲	۲	New Zealand
٥	٥	0	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	۲	Italy
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	۲	Scotland
0	٥	0	٥	0	٥	٥	0	0	0	٥	٥	0	0	0	0	_	۲	۲	۲	۲	Armenia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		۲				Moldova, Rep. of
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	۲				Cyprus
		0	0	0	0	0	0	0	0	0	0	0	0	0	0						Norway
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	_					Lebanon
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Macedonia, Rep. of
				0	0	0	0	0	0	0	0	0	0	0	0	-					Jordan
				9	9	0	0	0	0	0	0	0	0	0	0						Favot
						0	0	0	0	0	0	0	0	0	0		•				Egypt
•	•		•					0	0	0	0	0	0	0	0		•				Robroin
•	•		•	•				9	9	0	0	0	0	0	0			•	•		
•	•		•							5	5	0	0	0	0		•				Palestinian Nat'l Auth
•		•		•								0	0	0	0			•	•	•	Philippines
•					•		۲					0	0	0	0						Morocco
												0	0	0	0						Chile
	۲		۲		۲	۲		۲		۲			0	0	0		۲				Botswana
$\overline{\mathbf{x}}$									$\overline{\mathbf{O}}$			$\overline{\mathbf{v}}$	-	0	0						Saudi Arabia
۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		0		۲	۲	۲	۲	South Africa
				$\overline{\mathbf{v}}$										$\overline{\mathbf{v}}$				$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$		Ghana
																					Benchmarking Participants
٥	0	٥	0	٥	٥	0	٥	٥	٥	٥	0	0	٥	٥	٥			$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	Basque Country, Spain
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0			۲	Indiana State, US
٥	0	٥	0	0	0	0	0	٥	0	0	0	0	0	0	0		0				Ontario Province. Can.
٥	0	0	0	0	0	0	0	0	٥	0	0	٥	0	0	0		0	0	0		Quebec Province, Can.
									-			-									

• Average achievement significantly higher than comparison country

Average achievement significantly lower than comparison country

 $\overline{\mathbf{v}}$ 

Note: 5% of these comparisons would be statistically significant by chance alone.

TIMSS & PIRLS INTERNATIONAL STUDY CENTER, LYNCH SCHOOL OF EDUCATION, BOSTON COLLEGE

MATHEMATICS

Grade

0

0

MATHEMATICS

Grade

## Exhibit 2.2: Multiple Comparisons of Average Mathematics Achievement for Knowing Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Countries	Singapore	Hong Kong, SAR	Chinese Taipei	Japan	Belgium (Flemish)	England	Netherlands	United States	Lithuania	Hungary	Latvia	Italy	Russian Federation	Australia	Moldova, Rep. of	Cyprus	New Zealand	Scotland	Slovenia	Norway	Armenia	Iran, Islamic Rep. of	Philippines	Morocco	Tunisia		Indiana State, US	Ontario Province, Can.	Quebec Province, Can.
Singaporo		0	0	0	0	^	0	^	0	0	^	0	^	0	0	^	0	^	^	0	^	0	0	0	0		0	0	~
Hong Kong, SAR		•	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
Chinese Tainei	•				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
lanan	•	•			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	٥	0		0	0	0
Belgium (Elemish)			۲	۲		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
England	♥								0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0			0	0
Netherlands	۲	۲	۲	۲	۲				٥	0	٥	٥	0	٥	0	0	0	0	٥	٥	٥	٥	٥	٥	0		۲	0	0
United States	$\overline{\mathbf{v}}$	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\bullet}$				٥	0	0	0	٥	٥	0	٥	٥	٥	٥	0	٥	0	0	٥	0		$\odot$	0	٥
Lithuania	۲	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲						٥	0	0	0	٥	0	0	٥	0	0	٥	0		$\odot$		0
Hungary	$\overline{\bullet}$	$\odot$	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare						٥	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		$\odot$		٥
Latvia	۲	۲	۲	۲	۲	۲	۲	۲						٥	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲		٥
Italy	lacksquare	۲	۲	۲	۲	lacksquare	۲	lacksquare						٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲		٥
Russian Federation	۲	۲	۲	۲	۲	۲	۲	۲								٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲		
Australia	lacksquare	lacksquare	lacksquare	lacksquare	۲	lacksquare	lacksquare	lacksquare	lacksquare	$\odot$	$\odot$	$\odot$						٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	
Moldova, Rep. of	۲	۲	۲	۲	۲	۲	lacksquare	۲	۲	۲	$\odot$	۲						٥	٥	٥	٥	٥	٥	٥	0		۲		
Cyprus	lacksquare	۲	۲	۲	۲	lacksquare	lacksquare	lacksquare	lacksquare	$\odot$	$\odot$	$\odot$	lacksquare				٥	٥	٥	٥	٥	٥	٥	٥	٥		$\odot$	۲	
New Zealand	۲	۲	۲	۲	۲	۲	۲	lacksquare	۲	۲	۲	۲	۲			۲		٥	٥	0	٥	0	0	٥	0		۲	۲	۲
Scotland	۲	۲	۲	۲	۲	۲	۲	lacksquare	۲	۲	lacksquare	۲	۲	lacksquare	۲	lacksquare	۲		٥	٥	٥	٥	٥	٥	٥		۲	۲	۲
Slovenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		0	٥	0	0	٥	0		۲	۲	۲
Norway	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			٥	0	٥	٥		۲	۲	۲
Armenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			0	0	٥	0		۲	۲	۲
Iran, Islamic Rep. of	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	٥	0		۲	۲	
Philippines	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	0	_	۲	۲	۲
Morocco	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			۲	۲	۲	۲	۲	۲	۲		0				
Tunisia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			۲			۲	۲	۲
Benchmarking Participants					_	_					_	_	_		_	_		_			_	_	_		_	_	_		_
Indiana State, US		۲	۲				0	0	٥	٥	٥	٥	٥	٥	٥	0	0	0	0	0	٥	٥	٥	٥	0			0	0
Ontario Province, Can.	۲	۲	۲	۲	۲	۲	۲	۲						٥		0	0	0	0	0	٥	٥	٥	٥	0		۲		
Quebec Province, Can.	۲	۲	۲	۲	۲	۲	۲	$\overline{\bullet}$	$\overline{\mathbf{v}}$	۲	۲	۲					0	0	0	0	0	0	0	0	0		۲		

• Average achievement significantly higher than comparison country

 Average achievement significantly lower than comparison country

no significant difference in performances; and a circle with a triangle pointing down indicates significantly lower performance.

At the eighth grade, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had significantly higher achievement in the knowing domain than the other participating countries. With the exception of those four top-performing countries, Japan had significantly higher achievement than all the rest of the participating countries. Estonia, Belgium (Flemish), and Hungary also performed very well as did the Canadian province of Quebec, being outperformed by only the five top-scoring Asian countries.

At the fourth grade, Singapore had the highest average achievement in the knowing domain followed by Hong Kong SAR and then Chinese Taipei and Japan. Belgium (Flemish) outperformed all the participating countries except the four top-scoring Asian countries. England, the Netherlands, the United States, and the US state of Indiana also had higher average achievement than many of the other participating countries.

### Applying Knowledge and Conceptual Understanding

Exhibit 2.3 presents the distribution of student mathematics achievement in the cognitive domain of applying at the eighth (first page) and fourth (second page) grades.

At the eighth grade, led by Singapore, 24 countries and the four benchmarking participants had achievement in the applying domain significantly higher than the international average. Romania, Bulgaria, Norway, and Serbia performed no differently than the international average and 18 countries performed significantly below this average. At the fourth grade, also led by Singapore, 14 countries and the US state of Indiana had achievement significantly higher than the international average, two countries (Italy and Australia) and the two Canadian provinces had achievement similar to the international average, and 9 countries had achievement below it.

#### **TIMSS2003**

#### Exhibit 2.3: Distribution of Mathematics Achievement for Applying Cognitive Domain



Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Bevelopment Index**
Singapore	8	14.3		611 (3.6)	0.884 <sup>^</sup> pn
<sup>†</sup> Hong Kong, SAR	8	14.4		584 (3.2)	<sup>م</sup> 888.0
🍽 Korea, Rep. of	8	14.6		584 (2.2)	0.879
Chinese Taipei	8	14.2		582 (4.6)	
Japan	8	14.4		564 (2.2)	0.932
<sup>†</sup> Netherlands	8	14.3		543 (3.7)	0.938
Belgium (Flemish)	8	14.1		536 (2.7)	0.937
Estonia	8	15.2	and a second	528 (2.9)	0.833 <sup>[]</sup>
Hungary	8	14.5		523 (3.4)	0.837
Malaysia	8	14.3	the second se	512 (4.4)	0.790 .00
Australia	8 or 9	13.9		508 (4.8)	0.939
<sup>†</sup> Scotland	9	13.7	and the second se	505 (3.9)	0.930 <sup>±</sup>
Sweden	8	14.9	and the second	505 (2.8)	0.941
Latvia	8	15.0		504 (3.4)	0.811
Russian Federation	7 or 8	14.2		503 (3.7)	0.779
Slovak Republic	8	14.3		502 (3.7)	0.836
<sup>‡</sup> United States	8	14.2	and a second	502 (3.4)	0.937
<sup>1</sup> Lithuania	8	14.9		499 (2.8)	0.824
New Zealand	8.5 - 9.5	14.1		497 (5.3)	0.917
<sup>2</sup> Israel	8	14.0	and a second	495 (3.6)	0.905
Slovenia	7 or 8	13.8	and the second	491 (2.3)	0.881
Italy	8	13.9		484 (3.2)	0.916
Armenia	8	14.9	and the second se	478 (3.0)	0.729
Romania	8	15.0		475 (5.0)	0.773
Bulgaria	8	14.9		471 (4.7)	0.795
Norway	7	13.8		468 (2.7)	0.944
<sup>1</sup> Serbia	8	14.9		467 (2.9)	-
International Avg.	8	14.5		467 (0.5)	-
Moldova, Rep. of	8	14.9	· · · · ·	457 (3.9) 💿	0.700
Cyprus	8	13.8		457 (1.6) 💿	0.891
<sup>2</sup> Macedonia, Rep. of	8	14.6	and the second se	428 (3.8) 💿	0.784
Lebanon	8	14.6		426 (3.3) 💿	0.752
Jordan	8	13.9	and the second sec	422 (4.2) 💿	0.743
Tunisia	8	14.8		419 (2.3) 💿	0.740
Iran, Islamic Rep. of	8	14.4		416 (2.5) 💿	0.719
<sup>1</sup> Indonesia	8	14.5		408 (4.9) 💿	0.682
Egypt	8	14.4	and a second	404 (3.4) 💿	0.648
Bahrain	8	14.1		398 (1.6) 💿	0.839
Chile	8	14.2	and a second	391 (3.3) 💿	0.831
Palestinian Nat'l Auth.	8	14.1	and a second	388 (3.2) 💿	0.731
<sup>1</sup> <sup>‡</sup> Morocco	8	15.2		384 (2.9) 💿	0.606
Philippines	8	14.8		378 (4.8) 💿	0.751
Botswana	8	15.1		369 (2.7) 💿	0.614
Saudi Arabia	8	14.1		338 (3.6) 💿	0.769
Ghana	8	15.5	and the second sec	293 (4.0) 💿	0.567
South Africa	8	15.1		269 (5.3) 💿	0.684
<sup>‡</sup> England	9	14.3		503 (4.8)	0.930
<b>Benchmarking Participants</b>					
Basque Country, Spain	8	14.1		481 (2.3)	-
Indiana State, US	8	14.5		507 (5.9)	_
Ontario Province, Can.	8	13.8		522 (3.0)	-
Quebec Province, Can.	8	14.2		545 (3.0)	_
			0 100 200 300 400 500 600 700 800	Country average sign	nificantly higher

Country average significantly higher than international average

Country average significantly lower than international average

\* Represents years of schooling counting from the first year of ISCED Level 1.

\*\* Taken from United Nations Development Programme's Human Development Report 2003,

- p. 237-240.
- † Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- \* Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- 2 National Defined Population covers less than 90% of National Desired Population (see Exhibit C.1).
- ➡ Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
- () Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
- A dash (-) indicates comparable data are not available.

75th 95th

95% Confidence Interval for Average (±2SE)

5th 25th

#### Distribution of Mathematics Achievement for Applying Cognitive Domain Exhibit 2.3:

### **TIMSS2003** MATHEMATICS Grade

Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
Singapore	4	10.3		595 (5.9)	0.884
<sup>†</sup> Hong Kong, SAR	4	10.2		577 (3.3)	0.889
Japan	4	10.4		566 (2.1)	0.932
Chinese Taipei	4	10.2		561 (1.9)	
Belgium (Flemish)	4	10.0		546 (2.1)	0.937
Latvia	4	11.1		545 (3.3)	0.811
Russian Federation	3 or 4	10.6		542 (4.7)	0.779
<sup>1</sup> Lithuania	4	10.9		542 (2.9)	0.824
† Netherlands	4	10.2		541 (2.6)	0.938
Hungary	4	10.5		530 (3.4)	0.837
<sup>†</sup> England	5	10.3	and the second se	526 (4.1)	0.930
Cyprus	4	9.9	and the second	510 (2.8)	0.891
Moldova, Rep. of	4	11.0		507 (4.8)	0.700
<sup>†</sup> United States	4	10.2		505 (2.6)	0.937
International Avg.	4	10.3		495 (0.7)	- +
Italy	4	9.8		494 (3.6)	0.916
<sup>†</sup> Australia	4 or 5	9.9		490 (3.8)	0.939
<sup>†</sup> Scotland	5	9.7		487 (3.5) 💿	0.930
New Zealand	4.5 - 5.5	10.0		486 (2.3) 💿	0.917
Slovenia	3 or 4	9.8		477 (2.8) 💿	0.881
Armenia	4	10.9		462 (3.2) 💿	0.729
ø Norway	4	9.8		446 (2.2) 💿	0.944
Iran, Islamic Rep. of	4	10.4		391 (3.8) 💿	0.719
Philippines	4	10.8		364 (7.5) 💿	0.751
Morocco	4	11.0		349 (4.5) 💿	0.606
Tunisia	4	10.4		348 (4.6) 💿	0.740
Benchmarking Participants					
Indiana State, US	4	10.5		523 (3.3)	-
Ontario Province, Can.	4	9.8		498 (4.5)	-
Quebec Province, Can.	4	10.1		498 (2.7)	-
			0 100 200 300 400 500 600 700 800 ———————————————————————————————————	<ul> <li>Country average sign than international av</li> <li>Country average sign</li> </ul>	ificantly higher erage ificantly lower

 Country average significantly lower than international average

\* Represents years of schooling counting from the first year of ISCED Level 1.

- \*\* Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
- Met guidelines for sample participation rates only after replacement schools were included (see t Exhibit C.2).
- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
- () Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (-) indicates comparable data are not available.

# Exhibit 2.4: Multiple Comparisons of Average Mathematics Achievement for Applying Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Countries	Singapore	Hong Kong, SAR	Korea, Rep. of	Chinese Taipei	Japan	Netherlands	Belgium (Flemish)	Estonia	Hungary	Malaysia	Australia	Scotland	Sweden	Latvia	England	Russian Federation	Slovak Republic	United States	Lithuania	New Zealand	Israel	Slovenia	Italy	Armenia	Romania	Bulgaria	Norway	Serbia	Moldova, Rep. of	Cyprus
Singapore		0	٥	0	0	0	0	0	٥	٥	0	0	0	٥	٥	0	0	0	٥	0	٥	0	0	٥	0	0	٥	0	0	0
Hong Kong, SAR	۲				٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥
Korea, Rep. of	۲				0	0	0	0	٥	٥	٥	0	0	٥	٥	٥	٥	٥	0	0	0	0	0	0	0	0	٥	0	0	٥
Chinese Taipei	۲				0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	0	٥	٥	0	٥	٥
Japan	۲	۲	۲	۲		٥	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Netherlands								0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Belgium (Flemish)						~			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Estonia										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hungary										0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Malaysia	•	•	•	•	•	•	•	•	•										0	0	0	0	0	0	0	0	0	0	0	0
Scotland	•	•	•	•	•	•	•	•	•												•	0	0	0	0	0	0	0	0	0
Sweden	•	•	•	•	•	•	•	•	•												٥	0	0	0	0	0	0	0	0	0
Latvia				•	•			•													-	0	0	0	0	0	0	0	0	0
England																						0	0	0	0	0	0	0	0	0
Russian Federation	$\overline{\mathbf{v}}$	♥	$\overline{\bullet}$	$\overline{\bullet}$	♥	♥	♥	♥	♥													٥	٥	٥	٥	٥	٥	٥	٥	٥
Slovak Republic	۲	۲	۲	۲	۲	۲	۲	۲	۲													0	0	0	٥	0	٥	0	٥	٥
United States	۲	۲	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\mathbf{v}}$	lacksquare	lacksquare	lacksquare	۲													٥	٥	٥	٥	٥	٥	٥	٥	٥
Lithuania	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲												٥	٥	٥	٥	٥	٥	0	٥	٥
New Zealand	۲	۲	۲	lacksquare	۲	♥	♥	۲	۲	lacksquare													٥	٥	٥	٥	٥	٥	٥	٥
Israel	۲	۲	۲	۲	۲	♥	♥	۲	۲	۲	۲		۲										٥	٥	٥	٥	٥	0	٥	٥
Slovenia	۲	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	٥	0	٥	٥
Italy	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	0	٥	٥
Armenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	0
Romania	۲		۲	۲	۲	۲	۲	۲	•	•	۲	۲	۲	۲	۲	۲		۲		۲	۲	۲	0						0	0
Bulgaria																								0					0	0
Norway																													0	0
Serbia																														0
Moldova, Rep. of																														
Macadonia Ron of	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•		•	•	•	•	•	•	•	•	•	•	•		
	•				•	•	•			•		•		•	•	•		•		•			•		•		•			
lordan	•	•	•	•	•	•		•		•			•		•	•	•	•	•	•	•	•		•	•	•		•	•	•
Tunisia				•	•							•	•						•	•	•	•	•	•		•		•		
Iran, Islamic Rep. of	۲	۲	۲	۲	۲			۲					۲		۲	۲	۲	۲			۲	۲				۲			$\bigcirc$	
Indonesia	۲	۲	$\overline{\bullet}$	$\overline{\bullet}$	$\overline{\bullet}$	♥	$\overline{\bullet}$	$\overline{\bullet}$		$\bigcirc$	۲	$\overline{\bullet}$	$\overline{\bullet}$	lacksquare	$\overline{\bullet}$	$\bigcirc$	۲	$\bigcirc$	$\overline{\bullet}$	lacksquare	$\overline{\bullet}$	$\bigcirc$	$\bigcirc$							
Egypt	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Bahrain	۲	۲	۲	lacksquare	lacksquare	♥	lacksquare	lacksquare	۲	lacksquare	۲	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	۲	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	lacksquare	۲	۲
Chile	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Palestinian Nat'l Auth.	lacksquare	۲	۲	۲	♥	♥	♥	۲	۲	lacksquare	۲	♥	۲	۲	۲	lacksquare	۲	۲	۲	۲	۲	۲	♥	۲	♥	۲	۲	۲	۲	۲
Morocco	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Philippines	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Botswana	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲
Saudi Arabia						۲	۲			•		۲											۲							
Ghana																														
South Africa			$\mathbf{\bullet}$	$\bullet$															$\bullet$					$\bullet$				$\bullet$		
Benchmarking Participants	0	0	6	0	6	0	0	6	6	6	6	0	6	6	6	0	0	6	0	0	0	6					•	•	•	•
Basque Country, Spain																							~	•	•	~	0	0	0	0
Indiana State, US								J	۲	•	•	0	0	•	•	0	0	•	•	0	0	0	0	0	0	0	0	0	0	0
Ontario Province, Can.	•			•	•	J	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Quebec Province, Can.	U	U	J	J	J		9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9

Note: 5% of these comparisons would be statistically significant by chance alone.

MATHEMATICS

Grade

C

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# Exhibit 2.4: Multiple Comparisons of Average Mathematics Achievement for Applying Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Macedonia, Rep. of	Lebanon	Jordan	Tunisia	Iran, Islamic Rep. of	Indonesia	Egypt	Bahrain	Chile	Palestinian Nat'l Auth.	Morocco	Philippines	Botswana	Saudi Arabia	Ghana	South Africa		Basque Country, Spain	Indiana State, US	Ontario Province, Can.	Quebec Province, Can.	Countries
٥	٥	0	0	٥	٥	0	0	0	٥	٥	٥	٥	0	٥	٥		0	0	٥	0	Singapore
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥	٥	Hong Kong, SAR
0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥	٥	Korea, Rep. of
0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥	٥	Chinese Taipei
0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	0	0	٥	٥	٥		٥	٥	٥	٥	Japan
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0		Netherlands
0	0	0	٥	0	0	0	0	٥	0	٥	٥	٥	٥	0	٥	_	0	0	0	۲	Belgium (Flemish)
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0		۲	Estonia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0	0		۲	Hungary
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0		۲		Malaysia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0		۲		Australia
0	0	0	0	0	0	0	0	٥	0	0	0	0	٥	٥	0		0		۲		Scotland
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0		۲		Sweden
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0		۲		Latvia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0		۲		England
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0				Russian Federation
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0		۲		Slovak Republic
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0		۲		United States
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0		۲		Lithuania
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0				New Zealand
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	0	_	۲		Israel
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0				Slovenia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Italy
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Armenia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Romania
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0				Bulgaria
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	_					Norway
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Serbia
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Moldova, Rep. of
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Cyprus
			0	0	0	0	0	0	0	0	0	0	0	0	0	_					Macedonia, Rep. of
				0	0	0	0	0	0	0	0	0	0	0	0						Lebanon
0					0	0	0	0	0	0	0	0	0	0	0						Jordan
	e				0	0	0	0	0	0	0	0	0	0	0						Tunisia
						0	0	0	0	0	0	0	0	0	0						Iran, Islamic Kep. of
							9	0	0	0	0	0	0	0	0						Fruet
					0			0	0	0	0	0	0	0	0						Egypt
									9	9	0	0	0	0	0						Banrain
		•			•	•					9	0	0	0	0			•	•		Crille Balactinian Natil Auth
												0	0	0	0						Palestinian Nat'i Auth.
•	•		•	•								9	0	0	0				•	•	Rhilipping
	•		•	•									0	0	0				•	•	Rotewapa
•	•	•	•	•	•	•	•	•	•	•			9	0	0			•	•	•	Soudi Arabia
•			•		•					•		•		5	0		•				Ghana
•	•	•	•	•	•	•	•	•	•	•	•	•	•		9			•	•	•	
J	J	J	J	J		J	J	J	J	J	J	J	J	. U			J	J	. U		
^	^	^	^	^	0	^	^	^	^	^	^	0	^	^	^			ē	ē	e	Benchmarking Participants
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0				basque Country, spain
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	J	•	Ontario Brovince Car
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	^		Ontario Province, Can.
0	9	0	0	0	0	0	0	0	0	0	9	0	0	0	0		0	0	0		Quebec Province, Can.

• Average achievement significantly higher than comparison country

 Average achievement significantly lower than comparison country

Note: 5% of these comparisons would be statistically significant by chance alone.

TIMSS & PIRLS INTERNATIONAL STUDY CENTER, LYNCH SCHOOL OF EDUCATION, BOSTON COLLEGE

Grade (O

SOURCE: IEA's Trends in International Mathematics and Science Study (TIMSS) 2003

**Cognitive Domain** 

Exhibit 2.4:

## MATHEMATICS Grade

Multiple Comparisons of Average Mathematics Achievement for Applying

Instructions: Read across the row fo whether the average achievement c that of the comparison country, or if	or a c of the f the	coun e co re is	try 1 untr no	to co y in stati	omp the istic	are row ally	perf / is s sigr	orm signi nific	ianc ifica ant	e wi ntly diffe	ith t Iow eren	he c /er t ce b	oun han etw	tries tha een	s list t of the	ted a the ave	alon com rage	ig th ipar e ac	e to ison hiev	p of cou eme	f the intry ent c	e cha /, sig of th	art. 1 gnifi ie tv	The can vo c	syml tly h coun	ools ighe tries	ind er th	icate an	TIMASS) 2002
Countries	Singapore	Hong Kong, SAR	Japan	Chinese Taipei	Belgium (Flemish)	Latvia	<b>Russian Federation</b>	Lithuania	Netherlands	Hungary	England	Cyprus	Moldova, Rep. of	United States	Italy	Australia	Scotland	New Zealand	Slovenia	Armenia	Norway	Iran, Islamic Rep. of	Philippines	Morocco	Tunisia		Indiana State, US	Ontario Province, Can.	Quebec Province, Can.
Singapore		0	٥	0	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥		٥	0	0
Hong Kong, SAR	$\overline{\bullet}$		٥	٥	٥	٥	٥	0	٥	٥	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	<b>o</b> i <sup>+</sup>
Japan	۲	۲			0	٥	0	0	0	0	٥	0	0	٥	٥	0	٥	٥	٥	٥	0	٥	٥	٥	٥		٥	0	0
Chinese Taipei	$\overline{\bullet}$	۲			٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	0
Belgium (Flemish)	۲	۲	۲	۲						٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	0	<b>O</b> i
Latvia	lacksquare	$\overline{\mathbf{v}}$	lacksquare	$\odot$						0	٥	0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	0
Russian Federation	۲	۲	۲	۲						0	٥	0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	0	0
Lithuania	lacksquare	$\overline{\mathbf{v}}$	lacksquare	$\odot$						0	٥	0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	0
Netherlands	۲	۲	۲	۲						0	٥	0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	0	0
Hungary	lacksquare	$\overline{\mathbf{v}}$	lacksquare	$\odot$	lacksquare	$\odot$	lacksquare	lacksquare	lacksquare			0	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			٥	0
England	۲	۲	۲	۲	۲	۲	۲	۲	۲			0	0	٥	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			0	0
Cyprus	lacksquare	۲	۲	۲	۲	۲	lacksquare	lacksquare	۲	۲	lacksquare				٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	٥	٥
Moldova, Rep. of	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲				٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲		
United States	lacksquare	۲	۲	۲	۲	۲	lacksquare	lacksquare	۲	۲	lacksquare				٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥		۲		
Italy	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	٥	٥	٥	٥		۲		
Australia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	٥	٥	0	٥		۲		
Scotland	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	٥	٥	٥	٥		۲		۲
New Zealand	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					٥	٥	٥	٥	٥	0	٥		۲	۲	$\bigcirc$
Slovenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	٥	٥	٥	٥	٥		۲	۲	۲
Armenia	lacksquare	۲	۲	۲	۲	۲	۲	lacksquare	۲	۲	lacksquare	۲	۲	۲	lacksquare	۲	۲	۲	۲		٥	٥	٥	٥	٥		۲	۲	$\odot$
Norway	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	٥	٥	٥		۲	۲	۲
Iran, Islamic Rep. of	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	٥	٥		۲	۲	$\bigcirc$
Philippines	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	♥	♥	۲	۲	۲	♥					۲	۲	۲
Morocco	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					۲	۲	۲
Tunisia	lacksquare	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲					۲	۲	۲
Benchmarking Participants																													
Indiana State, US	۲	۲	♥	۲	♥	۲	۲	۲	۲			٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			٥	0
Ontario Province, Can.	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲						٥	٥	٥	٥	٥	٥	٥	٥		۲		
Quebec Province, Can.	۲	۲	♥	۲	♥	۲	۲	۲	۲	۲	۲	♥					٥	0	٥	٥	٥	٥	٥	٥	٥		۲		

Average achievement significantly higher than ٥ comparison country

Average achievement  $\overline{\mathbf{v}}$ significantly lower than comparison country

Exhibit 2.4 shows for the eighth (first two pages) and fourth (third page) grades how a country's average mathematics achievement in the applying domain compares to achievement in the other participating countries.

At the eighth grade, Singapore had the highest achievement in the applying domain. Hong Kong SAR, the Republic of Korea, and Chinese Taipei all performed equally well, but not as well as Singapore. These countries were followed by Japan with performance below that of the four top-scoring countries, but with significantly higher achievement in this domain than all of the other participants at the eighth grade. The Netherlands and Belgium (Flemish) were outperformed by the five top-scoring Asian countries, but also did very well.

At the fourth grade, results for the four Asian countries in the applying domain were nearly the same as for the knowing domain. The Singaporean students had the highest average achievement in the applying domain, followed by the students in Hong Kong SAR, who (with the exception of Singapore) had significantly higher achievement than students in the other participating countries. Japan and Chinese Taipei performed similarly to each other and had the next highest achievement after Hong Kong SAR. Compared to the knowing domain, however, several more countries performed similarly to each other and were in the second highest achieving group of countries. Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands had significantly higher achievement in the applying domain than the rest of the participating countries and benchmarking entities.

### Reasoning

The first and second pages of Exhibit 2.5 show the distribution of student mathematics achievement in the cognitive domain of reasoning at the eighth and fourth grades, respectively. Exhibit 2.6 shows, for the eighth and fourth grades (first two pages and third page, respectively), how a

#### TIMSS2003

#### Exhibit 2.5: Distribution of Mathematics Achievement for Reasoning Cognitive Domain



Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution	Average Scale Score	Human Development Index**
Singapore	8	14.3		583 (3.5)	0.884
🍽 Korea, Rep. of	8	14.6		582 (1.7) 🛛	0.879
Chinese Taipei	8	14.2		576 (4.2)	
Japan	8	14.4		576 (1.8)	0.932
<sup>†</sup> Hong Kong, SAR	8	14.4		569 (3.1)	0.889
† Netherlands	8	14.3		541 (3.8) 🗅	0.938 ÷
Belgium (Flemish)	8	14.1		533 (2.8)	0.937
Hungary	8	14.5		529 (3.1)	0.837
Estonia	8	15.2		523 (3.0)	0.833
Australia	8 or 9	13.9		515 (4.0)	0.939 .9
<sup>†</sup> Scotland	9	13.7		513 (3.4)	0.930
New Zealand	8.5 - 9.5	14.1		509 (5.2)	0.917
Sweden	8	14.9		508 (3.3)	0.941
+ United States	8	14.2		505 (3.3)	0.937
Slovak Republic	8	14.3		504 (3.2)	0.836
Malaysia	8	14.3		503 (3.4)	0.790
Latvia	8	15.0		500 (3.4)	0.811
Russian Federation	7 or 8	14.2		496 (3.6)	0.779
Slovenia	7 or 8	13.8		494 (2.5)	0.881
' Lithuania	8	14.9		489 (2.6)	0.824
Italy	8	13.9		489 (2.9)	0.916
<sup>2</sup> Israel	8	14.0		483 (3.3)	0.905
Norway	7	13.8		479 (2.8)	0.944
Bulgaria	8	14.9		471 (3.9)	0.795
Armenia	8	14.9		468 (2.8)	0.729
<sup>1</sup> Serbia	8	14.9		468 (2.6)	-
International Avg.	8	14.5		467 (0.5)	-
Romania	8	15.0		458 (4.5)	0.773
Cyprus	8	13.8		455 (1.7)	0.891
Moldova, Rep. of	8	14.9		453 (4.0)	0.700
<sup>2</sup> Macedonia, Rep. of	8	14.6		438 (3.7)	0.784
Jordan	ð	13.9		433 (3.7)	0.743
Banrain	8	14.1		424 (2.2)	0.839
Iran, Islamic Rep. of	8	14.4		417 (2.8)	0.719
Chile	8	14.0		410 (3.0)	0.752
	8	14.2		409 (3.5)	0.831
Balastinian Nat'l Auth	0	14.5		406 (4.3)	0.002
Falestinian Nat i Autri.	0	14.1		404 (2.7)	0.731
Tupicio	0	14.4		200 (2.7)	0.040
	0	14.0		201 (2.2)	0.740
Bhilipping	0	14.9		259 (5.2)	0.808
Botswana	8	14.0		353 (3.7)	0.614
Saudi Arabia	8	14.1		248 (4 2) <b>•</b>	0.014
Chana	8	14.1		212 (4.0)	0.703
South Africa	8	15.5		287 (5.0)	0.507
<sup>‡</sup> England	9	14.3		509 (4.7)	0.004
Renchmarking Participants	5	C.F-1		505 (4.7)	0.550
Basque Country Spain	Q	14.1		494 (2 A)	_
Indiana State US	2 2	14.5		503 (5 2)	_
Ontario Province, Can	8	13.8		527 (3 0)	_
Ouebec Province, Can	8	14.2		539 (3.2)	-
	÷		0 100 200 300 400 500 600 700 800	(, •	

Country average significantly higher than international average

Country average significantly lower than international average

\* Represents years of schooling counting from the first year of ISCED Level 1.

- \*\* Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
- † Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- \* Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- Did not satisfy guidelines for sample participation rates (see Exhibit C.2).
- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- A dash (--) indicates comparable data are not available.

75th 95th

Percentiles of Performance

95% Confidence Interval for Average (±2SE)

5th 25th

2

...

National Defined Population covers less than 90% of National Desired Population (see Exhibit C.1). Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

TIMSS & PIRLS INTERNATIONAL STUDY CENTER, LYNCH SCHOOL OF EDUCATION, BOSTON COLLEGE

#### Exhibit 2.5: Distribution of Mathematics Achievement for Reasoning Cognitive Domain

n	Grade

**TIMSS2003** 

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Countries	Years of Schooling*	Average Age	Mathematics Achievement Distribution Average Scale Score	Human Development Index**
Singapore	4	10.3	<b>5</b> 74 (6.1)	0.884 pn
<sup>†</sup> Hong Kong, SAR	4	10.2	<b>564 (3.7)</b>	0.889 g
Chinese Taipei	4	10.2	563 (2.2)	- je
Japan	4	10.4	562 (1.7)	0.932 <sup>30</sup> p
Belgium (Flemish)	4	10.0	541 (2.2)	0.937
<sup>†</sup> England	5	10.3	537 (3.5) O	0.930 ÷
<sup>†</sup> Netherlands	4	10.2	535 (2.9)	<b>0.938</b>
Latvia	4	11.1	<b>531 (3.2)</b>	0.811 <sup>늘</sup>
<sup>1</sup> Lithuania	4	10.9	<b>526 (3.1)</b>	0.824
<b>Russian Federation</b>	3 or 4	10.6	<b>526 (4.8)</b>	0.779 .j
Hungary	4	10.5	<b>524 (3.2)</b>	0.837
<sup>†</sup> United States	4	10.2	519 (2.5)	0.937
Cyprus	4	9.9	<b>516 (2.4)</b>	0.891 ····
<sup>†</sup> Australia	4 or 5	9.9	<b>5</b> 07 (3.6)	0.939 ja
New Zealand	4.5 - 5.5	10.0	503 (2.2)	0.917
Italy	4	9.8	499 (4.0)	0.916
<sup>†</sup> Scotland	5	9.7	498 (3.1)	<b>0.930</b>
International Avg.	4	10.3	495 (0.7)	
Moldova, Rep. of	4	11.0	494 (4.9)	0.700
Slovenia	3 or 4	9.8	485 (2.6) 💿	0.881
ø Norway	4	9.8	468 (2.1) 💿	0.944
Armenia	4	10.9	<b>445 (3.1)</b>	0.729
Iran, Islamic Rep. of	4	10.4	<b>4</b> 00 (3.4) 💿	0.719
Morocco	4	11.0	<b>368 (4.4)</b>	0.606
Philippines	4	10.8	<b>359</b> (7.4) 💿	0.751
Tunisia	4	10.4	<b>340 (4.2)</b>	0.740
Benchmarking Participants				
Indiana State, US	4	10.5	• 528 (3.4) <b>•</b>	-
Ontario Province, Can.	4	9.8	<b>523 (3.6)</b>	-
Quebec Province, Can.	4	10.1	• 512 (2.6) <b>O</b>	-
			0     100     200     300     400     500     600     700     800       Percentiles of Performance       5th     25th     75th     95th     Country average sign than international average sign than internatinternational average sign than international average sign	ificantly higher erage

95% Confidence Interval for Average (±2SE)

Country average significantly lower than international average

\* Represents years of schooling counting from the first year of ISCED Level 1.

- \*\* Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
- † Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
- 1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
- ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
- ( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
   A dash (-) indicates comparable data are not available.

MATHEMATICS Grade Ο

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# Exhibit 2.6: Multiple Comparisons of Average Mathematics Achievement for Reasoning Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Countries	Singapore	Korea, Rep. of	Chinese Taipei	Japan	Hong Kong, SAR	Netherlands	Belgium (Flemish)	Hungary	Estonia	Australia	Scotland	New Zealand	England	Sweden	United States	Slovak Republic	Malaysia	Latvia	Russian Federation	Slovenia	Lithuania	Italy	Israel	Norway	Bulgaria	Armenia	Serbia	Romania	Cyprus	Moldova, Rep. of
Singanoro					0	0	Δ	Δ	0	0	0	0	^	Δ	0	0	0	0	Δ	^	0	0	0	0	0	0	0	0	0	~
Koroa Rop of				^	0	0	~	~	~	0	~	0	~	~	0	0	~	~	0	0	0	0	0	0	0	0	~	~	0	0
Chiposo Taipoi					-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
lanan						٥	٥	0	0	0	0	0	٥	٥	٥	٥	0	0	0	٥	0	0	0	0	0	0	0	0	0	0
Hong Kong SAB	۲					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Netherlands			$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	$\overline{\mathbf{v}}$	-		٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	0
Belgium (Elemish)					۲			-	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hungary			•	•		$\overline{\mathbf{v}}$			-	0	0	0	٥	0	0	٥	0	0	0	٥	٥	0	0	0	٥	0	0	0	0	0
Estonia						•	$(\mathbf{v})$			-	0	0	0	٥	٥	٥	0	0	0	0	٥	0	0	0	٥	0	0	0	0	0
Australia			•	•				$\overline{\mathbf{v}}$			-	-	•	-	-	٥	0	0	0	٥	٥	0	0	0	٥	0	0	0	0	0
Scotland	•	•	•	•	•	•	۲	۲								0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
New Zealand	•	•	•	•												-	-	-	0	0	0	0	0	0	0	0	0	0	0	0
Fogland	•		•	•	•	•	•	۲	•										0	0	0	0	0	0	0	0	0	0	0	0
Sweden																			0	0	0	0	0	0	0	0	0	0	0	0
	•		•	•	•	•	•	۲	•											0	0	0	0	0	0	0	0	0	0	0
Slovak Bepublic	•	•	•	•																0	0	0	0	0	0	0	0	0	0	0
Malaysia	•																			0	0	0	0	0	0	0	0	0	0	0
Latvia																				•	0	0	0	0	0	0	0	0	0	0
Russian Edderation	•					•						$\bigcirc$	۲								-	-	0	0	0	0	0	0	0	0
Slovenia	•	•	•	•		•	•	•				•	•		$\overline{\mathbf{v}}$								0	0	0	0	0	0	0	0
Lithuania	•	•		•	•	•	•	•	•	•	•	•	•	•	•	•	•						-	0	0	0	0	0	0	0
Litituariia																								~	0	0	~	0	~	0
Italy	•					•	•	•	•		•				•	•		•						-	0	0	0	0	~	0
Norway	•																			•					U	0	0	0	~	0
Norway	•					•	•	•	•						•	•		•		•	•	•				•	•	0	~	0
Armonia																												0	~	0
Annenia	•					•	•	•							•	•												-	~	0
Serbia						•									•	•				•									•	•
Komania	•					•	•	•							•	•				•					•					
Cyprus						•									•	•				•					•					
Macadania Ran of	•					•	•	•							•	•				•					•					
Macedonia, Rep. of	•					•									•	•				•					•					
Jordan	•					•	•	•	•		•	•	•	•	•	•	•	•	•	•		•			•	•	•		•	
Banrain	•					•									•	•				•					•					
Iran, Islamic Rep. of	•					•	•	•	•		•	•	•	•	•	•	•	•	•	•		•			•	•	•		•	
Lebanon	•					•									•	•				•					•					
Chile																				•										
Indonesia Delectivity Nettle Aceth															•															
Palestinian Nat'l Auth.																				•										
Egypt																														
Tunisia																														
Morocco															•					•										
Philippines																														
Botswana		•			•	•	•	•	•	•	•	•	•	•	•	•	•	•		•		•	•	•	•		•	•		•
Saudi Arabia												•											•	•						
Ghana					•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•			•	•		•		•	•
South Africa		J	J	J	J	J	J	J	J	J	J	۲	J	J	J	U	J	J	J	J	J	J	J	۲	J	U	J	J	J	U
Benchmarking Participants	0	0	0	0	0	0	6	6	0	0	0	0	6	0	6	0	0						•	•	•	~	•	•	•	•
Basque Country, Spain																					•	•	0	0	0	0	0	0	0	0
Indiana State, US							۲	۲	۲	•					•	•				•	0	0	0	0	0	0	0	0	0	0
Ontario Province, Can.						۲		•	•	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Quebec Province, Can.	۲		۲	۲	۲			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

# Exhibit 2.6: Multiple Comparisons of Average Mathematics Achievement for Reasoning Cognitive Domain

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Macedonia, Rep. of	Jordan	Bahrain	Iran, Islamic Rep. of	Lebanon	Chile	Indonesia	Palestinian Nat'l Auth.	Egypt	Tunisia	Morocco	Philippines	Botswana	Saudi Arabia	Ghana	South Africa		Basque Country, Spain	Indiana State, US	Ontario Province, Can.	Quebec Province, Can.	Countries
0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥	٥	٥	٥	Singapore
٥	٥	0	٥	0	0	0	0	٥	0	0	٥	٥	0	0	٥		٥	0	0	0	Korea Ren of
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	Chinese Taipei
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	lanan
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0	0	Hong Kong, SAR
٥	٥	0	٥	0	٥	0	0	٥	0	0	٥	٥	0	0	٥		٥	٥	٥		Netherlands
٥	0	٥	0	٥	0	٥	٥	0	0	0	0	0	٥	0	٥		0	0			Belgium (Flemish)
٥	٥	٥	٥	٥	0	٥	٥	0	٥	٥	٥	٥	٥	0	٥		٥	٥		$\overline{\bullet}$	Hungary
٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥	0	0	٥	0	٥		٥	٥		۲	Estonia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥		$\overline{\bullet}$	$\overline{\bullet}$	Australia
٥	0	0	0	0	٥	0	0	0	0	0	0	0	0	0	٥		٥		۲	۲	Scotland
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	0	٥		٥		lacksquare	lacksquare	New Zealand
٥	٥	0	0	0	٥	0	0	0	٥	٥	0	0	0	0	٥		٥		۲	۲	England
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥		lacksquare	lacksquare	Sweden
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥		۲	۲	United States
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		٥		lacksquare	lacksquare	Slovak Republic
٥	٥	٥	٥	٥	٥	0	0	0	٥	٥	٥	٥	٥	0	٥		٥		lacksquare	۲	Malaysia
٥	٥	٥	٥	٥	٥	0	0	٥	٥	٥	٥	٥	٥	0	٥				lacksquare	lacksquare	Latvia
٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥				۲	۲	Russian Federation
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥				lacksquare	lacksquare	Slovenia
٥	٥	٥	٥	٥	٥	٥	٥	0	0	٥	٥	٥	٥	0	٥			۲	۲	♥	Lithuania
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			lacksquare	lacksquare	lacksquare	Italy
٥	٥	٥	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	0	٥		$\odot$	۲	۲	۲	Israel
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		lacksquare	lacksquare	۲	♥	Norway
٥	٥	0	٥	0	٥	0	0	0	٥	٥	٥	٥	0	0	٥		۲	۲	۲	۲	Bulgaria
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	۲	Armenia
٥	٥	٥	0	٥	٥	٥	٥	0	0	0	0	0	٥	0	٥		۲	۲	۲	۲	Serbia
٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥		۲	۲	۲	♥	Romania
٥	0	٥	0	٥	٥	٥	٥	0	0	0	0	0	٥	0	٥	_	۲	۲	۲	۲	Cyprus
٥	٥	٥	٥	٥	0	٥	٥	٥	٥	0	٥	٥	٥	0	٥		۲	۲	۲		Moldova, Rep. of
		0	0	0	0	0	0	0	0	0	0	0	0	0	0	_	۲	۲	۲	۲	Macedonia, Rep. of
		0	0	0	0	0	0	0	0	0	0	0	0	0	0		۲	۲	۲		Jordan
		_	0	٥	0	0	0	0	0	0	0	0	0	0	0	_					Bahrain
						0	0	0	0	0	0	0	0	0	0						Iran, Islamic Rep. of
								0	0	0	0	0	0	0	0						Lebanon
	•	•	e						9	0	0	0	0	0	0		•	•	•		Chile
				_						0	0	0	0	0	0						Indonesia
										9	0	0	0	0	0						Palestinian Nat'i Auth.
•											0	0	~	0	0		•		•		Egypt
•	•	•	•	•	•						Ň	Ň	Ň	Ň	~		•	•	•	•	Morocco
•	•	•	•	•	•	•	•				-	-	-	0	0		•	•	•		Philippines
•	•	•	•	•	•				•	•				0	0		•	•	•	•	Philippines
	•	•	•						•	•				0	0		•	•	•		Saudi Arabia
		۲			۲						۲	۲	۲	-	0						Ghana
																					South Africa
																					Den elementer - D
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0						Besque Country Spain
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0				•	•	Indiana State US
0	0	0	٥	٥	٥	٥	٥	٥	٥	0	٥	٥	0	٥	٥		٥	0			Ontario Province, Can
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		0	0	٥		Quebec Province, Can
-	-	-	-	-	-	-	-	-	-	-	-	-	-		-		-	-	-		quebec i toville, call.

• Average achievement significantly higher than comparison country

 Average achievement significantly lower than comparison country

Note: 5% of these comparisons would be statistically significant by chance alone.

MATHEMATICS

Grade

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MATHEMATICS

Grade

#### Exhibit 2.6: Multiple Comparisons of Average Mathematics Achievement for Reasoning **Cognitive Domain**

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

Instructions: Read across the row for whether the average achievement of that of the comparison country, or i	or a c of the f the	coun e co re is	untr no	to co y in stat	omp the istic	are row ally	perf / is s sigr	orm igni ifica	anc fica ant	e wi ntly diffe	ith t low eren	he c /er t ce b	oun han etw	trie tha een	s list t of the	ted a the ave	alon com rage	g th ipar e ac	e to ison hiev	p of cou eme	the intry ent c	cha , sig of th	art. 1 gnifi e tv	The s cant vo c	syml tly h oun	ools ighe tries	ind er th	icati Ian	a 000
Countries	Singapore	Hong Kong, SAR	Chinese Taipei	Japan	Belgium (Flemish)	England	Netherlands	Latvia	Lithuania	Russian Federation	Hungary	United States	Cyprus	Australia	New Zealand	Italy	Scotland	Moldova, Rep. of	Slovenia	Norway	Armenia	Iran, Islamic Rep. of	Morocco	Philippines	Tunisia		Indiana State, US	Ontario Province, Can.	Quebec Province, Can.
Singapore				0	0	0	0	٥	٥	٥	٥	0	٥	٥	٥	٥	0	٥	0	٥	٥	٥	٥	٥	٥		0	0	0
Hong Kong, SAR					٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	0	٥	٥	٥	٥	٥	٥		٥	٥	0
Chinese Taipei					0	٥	٥	٥	٥	0	٥	0	٥	٥	٥	0	0	٥	0	٥	٥	٥	0	٥	٥		٥	٥	0
Japan	۲				0	0	٥	٥	0	0	0	0	0	0	0	0	٥	0	0	٥	٥	0	0	٥	٥		0	0	0
Belgium (Flemish)	۲	۲	۲	۲				٥	0	0	0	0	0	0	٥	0	٥	0	٥	٥	٥	0	0	٥	٥		0	0	0
England	۲	۲	۲	۲					٥		٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥	٥			٥	0
Netherlands	۲	۲	۲	۲					0		0	0	0	0	٥	0	٥	0	٥	٥	٥	0	0	٥	0			0	0
Latvia	۲	۲	۲	۲	۲							٥	0	0	٥	0	٥	٥	٥	٥	٥	0	0	٥	٥				0
Lithuania	۲	۲	۲	۲	۲	۲	۲						٥	٥	٥	٥	0	٥	0	٥	٥	٥	0	٥	٥				0
Russian Federation	۲	۲	۲	۲	۲					_				0	0	0	٥	٥	0	٥	٥	0	0	٥	٥				0
Hungary	۲	۲	۲	۲	۲	۲	۲						0	0	٥	0	٥	0	٥	٥	٥	0	0	٥	٥				0
United States	۲	۲	۲	۲	۲	۲	۲	۲						٥	٥	0	٥	٥	0	٥	٥	٥	٥	٥	٥		۲		0
Cyprus	۲	۲	۲	۲	۲	۲	۲	۲	۲		۲			٥	٥	0	0	٥	0	٥	٥	0	0	٥	٥		۲		
Australia	۲	۲	۲	۲	۲		۲	۲	۲	۲	۲	۲	۲					0	0	٥	0	0	0	0	٥		۲	۲	
New Zealand	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲						0	٥	٥	0	0	٥	٥		۲	۲	۲
Italy	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	۲	۲	۲		_				0	٥	٥	0	0	٥	٥		۲	۲	۲
Scotland	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲						0	0	0	0	0	0	٥		۲	۲	۲
Moldova, Rep. of	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	۲	۲	۲	۲	_		_			٥	٥	0	0	٥	٥		۲	۲	۲
Slovenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			٥	٥	0	0	٥	0		۲	۲	۲
Norway	۲	۲	۲	۲	۲	$\overline{\bullet}$	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		٥	0	0	٥	٥		۲	۲	۲
Armenia	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		0	0	٥	٥		۲	۲	۲
Iran, Islamic Rep. of	۲	۲	۲			۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		0	0	0			۲	۲
Morocco	۲	۲	•			۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲		۲	۲	۲			0		۲	۲	۲
Philippines	۲	۲						۲	۲	۲	۲		۲	۲		۲	۲	۲		۲	۲	۲			0		۲		
Tunisia	۲					۲	۲	۲	۲	۲	۲		۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲	۲			۲	۲	
Benchmarking Participants						_																				_	_	_	
Indiana State, US	۲	۲	۲	۲	۲							0	٥	٥	0	٥	0	0	0	٥	٥	٥	٥	٥	٥				0
Ontario Province, Can.	۲	۲	۲	۲	۲	۲	۲							٥	0	٥	0	0	0	٥	٥	٥	٥	٥	٥				0
Quebec Province, Can.	۲	۲	۲	۲	۲		$\overline{\mathbf{v}}$	$\overline{\bullet}$	$\bigcirc$	۲	۲	۲			0	0	0	0	0	0	0	0	0	0	0		۲	۲	

0 Average achievement significantly higher than comparison country

Average achievement significantly lower than  $\bigcirc$ comparison country

country's average mathematics achievement in the reasoning domain compares to achievement in the other participating countries.

At the eighth grade, average achievement in the reasoning domain ranged from 583 in Singapore to 287 in South Africa. Twentyfour countries and the four benchmarking participants performed significantly above the international average, three countries (Bulgaria, Armenia, and Serbia) performed comparably to the international average, and 19 countries performed significantly below the average.

At the eighth grade, looking at both Exhibits 2.5 and 2.6, it can be seen that the rank ordering of significant differences in achievement is rather complicated for the reasoning domain. Singapore and the Republic of Korea had the highest average achievement in the reasoning domain, nearly identical (583 and 582), but Singapore had a larger standard error (3.5 to 1.7). Thus, the Republic of Korea had significantly higher achievement than every participating country except Singapore and Chinese Taipei whereas Singapore (with the larger standard error) had higher average achievement than every participating country except the Republic of Korea, Chinese Taipei, and Japan. Chinese Taipei and Japan had the same average score (576) followed by Hong Kong SAR. Chinese Taipei (also with a relatively larger standard error of 4.2) did not perform statistically differently than the other three Asian countries, whereas a difference was found between the Republic of Korea and Japan due to their small standard errors. Hong Kong SAR was outperformed only by Singapore and the Republic of Korea. The Netherlands and Belgium (Flemish) only were outperformed by the five top-scoring Asian countries.

At the fourth grade, performance ranged from 574 for Singapore to 340 for Tunisia. Fifteen countries and the three benchmarking participants performed significantly above the international average, three countries (Italy, Scotland, and Moldova) performed essentially at the international average, and seven countries performed significantly below the international average. Singapore had the highest achievement, outperforming all countries except Hong Kong SAR and Chinese Taipei. Hong Kong, Chinese Taipei and Japan had similar achievement followed by Belgium (Flemish), England, and the Netherlands (all with similar average achievement and only outperformed by the four highest-achieving Asian countries).

### **Overview Across Domains**

At both the eighth and fourth grades, the countries with the highest achievement in each of the three cognitive domains also tended to be the highest-scoring countries (though not always in the same rank order) on the overall mathematics assessment. At the eighth grade (see Exhibit 1.1), the four countries with the highest overall mathematics achievement were Singapore followed by the Republic of Korea, Hong Kong SAR, and Chinese Taipei (only outperformed by Singapore). Japan had the next highest achievement outperforming all the rest of the participating countries except the previous four countries. Belgium (Flemish), the Netherlands, Estonia, Hungary, and the Canadian province of Quebec also performed well (at least as well or better than all other participants except the five Asian countries listed above).

- In knowing, similar to overall mathematics achievement, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement followed by Japan (see Exhibits 2.1 and 2.2). As a slight difference compared to the results for overall mathematics achievement, the four top-scoring Asian countries performed similarly to each other in the knowing domain. Estonia, Belgium (Flemish), Hungary, and the Canadian province of Quebec were outperformed only by the five top-achieving Asian countries.
- In applying, Singapore had the highest average achievement followed by Hong Kong SAR, the Republic of Korea, and Chinese Taipei and then Japan (see Exhibits 2.3 and 2.4). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec were next (only outperformed by the five top-achieving Asian countries).

• In reasoning, Singapore and the Republic of Korea performed very similarly followed by Chinese Taipei and Japan and then Hong Kong SAR (see Exhibits 2.5 and 2.6). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec also had relatively high achievement, only being outperformed by the five Asian countries.

At the fourth grade, Singapore was the highest-performing country in overall mathematics followed by Hong Kong SAR, and then by Japan and Chinese Taipei who performed similarly (see Exhibit 1.1). Belgium (Flemish) had higher achievement than all countries except these four Asian countries.

- In knowing, the pattern at the fourth grade was the same as for overall mathematics (see Exhibits 2.1 and 2.2). The four Asian countries had the best achievement (Singapore followed by Hong Kong SAR, and then by Chinese Taipei and Japan) with Belgium (Flemish) having higher achievement than all countries except the four best-achieving Asian countries.
- In applying, the pattern for the four high-achieving Asian countries was the same as for overall mathematics (see Exhibits 2.3 and 2.4). However, Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands all followed, performing similarly to each other with lower achievement than the four Asian countries, but higher achievement than the rest of the participating countries.
- In reasoning, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement (see Exhibits 2.5 and 2.6). Japan had achievement similar to Hong Kong SAR and Chinese Taipei, but was outperformed by Singapore. Belgium (Flemish), England, and the Netherlands had achievement equal to or higher than all participants except the four top-achieving Asian countries.

Just as countries with high achievement on the mathematics assessment as a whole had high achievement in the three cognitive domains, countries scoring lowest on the assessment as a whole (e.g., South Africa, Ghana, and Saudi Arabia at the eighth grade and the Philippines, Morocco, and Tunisia at the fourth grade) also tended to have low performance in all three cognitive domains. For some middle-performing countries, however, performance was more varied. For example, at the eighth grade, Armenia performed above the international average in the knowing and applying domains (480 and 478) but essentially at the international average in the reasoning domain (468).

Looking at the range in scale scores across the cognitive domains at the eighth grade, the differences in average achievement between the highest- and lowest-performing countries were largest in the knowing domain (360 score points), next largest in the applying domain (342), and smallest in the reasoning domain (296). As described in the following sections, several more countries performed significantly above the international average in the knowing domain than in the applying and reasoning domains.

At the fourth grade, with fewer countries, the range in performance between the highest- and lowest-performing countries was smaller than at the eighth grade, but the pattern was similar. The largest difference was in the knowing domain (288), next in the applying domain (246), and the smallest difference was in the reasoning domain (234). In each of the three cognitive domains, about the same number of participants performed above, similar to, or below the international average.