## Chapter 2

## Mathematics Achievement in the Cognitive Domains at the Fourth and Eighth Grades

This chapter of the report presents the TIMSS 2003 mathematics achievement results for each of the three cognitive domains. Following the presentation of the results, for each domain in turn - knowing, applying, and reasoning - there is an overview of performance across domains.

## Knowing Facts, Procedures, and Concepts

The first page of Exhibit 2.1 presents the distribution of students' mathematics achievement in the cognitive domain of knowing facts, procedures, and concepts for the 46 countries and four benchmarking entities that participated in TIMSS 2003 at the eighth grade, and the second page presents the distribution of student achievement for the 25 countries and three benchmarking entities that participated at the fourth grade. Countries are shown in decreasing order of average (mean) scale score, together with an indication of whether the country average is significantly higher or lower than the international average. To provide a basis of comparison for the performance of each country in each cognitive domain, the international average across countries for each domain was scaled to be 467 , the same as the international average for mathematics overall. As explained in

Chapter 1 (footnote l), the benchmarking entities were not included in computing the international average. Also, as previously discussed in conjunction with Exhibit 1.1, the years of formal schooling and average age of the students in each country are shown to aid in interpretation of the achievement results. This information also is repeated in Exhibit 2.1 as well as in 2.3 and 2.5 for ease of reference.

Finally, as a reminder that not all countries are equally well equipped to meet the challenge of educating their young people, Exhibit 2.1 as well as Exhibits 2.3 and 2.5 include the value for each country on the Human Development Index provided by the United Nations Development Programme (see Human Development Report 2003). The index has a minimum value of 0 and a maximum of 1.0 . Countries with high values on the index enjoy long life expectancy, high levels of school enrollment and adult literacy, and a good standard of living as measured by per capita GDP. For example, at the eighth grade, TIMSS countries with index values greater than 0.9 included Australia, Belgium (Flemish), England, Israel, Italy, Japan, New Zealand, Norway, The Netherlands, Scotland, Sweden, and the United States. For all three cognitive domains, all of these countries (except Norway in the knowing and applying domains) had average achievement above the international average. However, not all countries performing above the overall international average in the three cognitive domains had an index value as high as 0.9 . Within each of the cognitive domains, the relationship between a country's index value and average student achievement was fairly similar.

As shown in Exhibit 2.1, in the knowing domain for the eighth grade, similar to overall mathematics performance, there was a wide range in performance between the highest- and lowest-performing countries, from 592 in the Republic of Korea to 232 in Ghana. Twentyseven countries and the four benchmarking entities performed above the international average and 17 countries scored below the international average. Moldova and Cyprus performed about the same as the international average.

At the fourth grade, the difference was also large between the highest-performing country Singapore (626) and the lowest-performing country Tunisia (338). Thirteen countries and the three benchmarking entities performed above the international average and eight countries performed below the international average. The four countries performing about at the international average were Australia, Moldova, Cyprus, and New Zealand.

For both the eighth and fourth grades, Exhibit 2.1 illustrates the broad range of achievement both within and across the countries assessed. It provides a graphical representation of student performance within each country. The bar graph for each country shows the 5 th, 25 th, 75 th, and 95 th percentiles ${ }^{1}$ as well as the $95 \%$ confidence for the mean. Each percentile point indicates the percentage of students below that point on the scale. For most TIMSS 2003 participants at the eighth grade, there was an enormous range within each country between the highest and lowest scores, often as much as 400 scale-score points. This range was as large or larger than the difference in mean achievement between the highest and lowest performing country. For the eighth grade knowing scale, the range for most students in the higher-achieving countries was from 400 to 700. In comparison, it tended to be between 300 and 600 for medium-performing countries and from 200 to 500 (or even lower) in the lower-performing countries.

Exhibit 2.2 shows how a country's average mathematics achievement in the knowing domain compares to achievement in the other participating countries. The results for the eighth grade are shown on the first two pages and for the fourth grade on the third page. The figure for each grade shows whether or not the differences in average achievement between pairs of countries are statistically significant. To read the table, select a country of interest from the first column and read across the row corresponding to that country. A circle with a triangle pointing up indicates significantly higher performance than the comparison country listed across the top; absence of a symbol indicates


* Represents years of schooling counting from the first year of ISCED Level 1 .
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
+ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
£ Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1)
2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1)

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

$\begin{array}{ll}\text { * } \quad \text { Represents years of schooling counting from the first year of ISCED Level } 1 . \\ \text { ** } \quad \text { Taken from United Nations Development Programme's Human Development Report 2003, } \\ \text { p. 237-240. } \\ \dagger \quad & \text { Met guidelines for sample participation rates only after replacement schools were included (see } \\ \text { Exhibit C.2). } \\ 1 & \text { National Desired Population does not cover all of International Desired Population (see Exhibit C.1). }\end{array}$
ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

| Exhibit 2.2: | Multiple Comparisons of Average Mathematics Achievement for Knowing <br> Cognitive Domain | мathemarcs <br> Grade |
| :--- | :--- | :--- |

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

| Countries |  | 高 |  |  |  |  |  | $\begin{aligned} & \text { on } \\ & \text { in } \\ & \hline \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| Korea, Rep. of |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Singapore |  |  |  |  | 0 | 0 | - | 0 | - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | $\bigcirc$ | - | 0 | 0 | 0 |  |
| Hong Kong, SAR |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Chinese Taipei |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 00 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Japan | - | - | - | - |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Estonia | - | - | - | - | - |  |  |  | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Belgium (Flemish) | - | - | - | - | - |  |  |  | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | $0 \cdot$ | - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Hungary | - | - | - | - | - |  |  |  | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Netherlands | - | - | - | - | - | - | - | - | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Russian Federation | - | - | - | $\bigcirc$ | - | - | - | - |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Latvia | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Slovak Republic | - | - | - | - | - | - | - | - | - |  |  |  |  |  | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Lithuania | $\bigcirc$ | - | - | $\bigcirc$ | - | - | - | - | - ${ }^{-}$ |  |  |  |  |  |  |  | 0 | - | $0 \cdot$ | 0 | 0 O | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| United States | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | - | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Malaysia | - | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ |  |  |  |  |  |  | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Israel | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Slovenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Australia | - | - | - | - | $\bigcirc$ | - | - | - | - - | - | - | - | - | - |  |  |  |  |  | 0 |  |  |  | 0 | 0 | 0 | 0 | 0 |  |
| Serbia | - | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - |  |  |  |  |  | 0 |  |  | 0 | 0 | 0 | 0 | 0 |  |
| England | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| Sweden | - | - | - | - | - | - | - | - | - $\square^{-}$ | - | - | - | - | - | - | - | - | - | $\bigcirc$ |  |  |  |  |  |  |  | 0 | 0 |  |
| Bulgaria | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Romania | $\bigcirc$ | - | - | - | - | - | - | - | - ${ }^{-}$ | $\cdot$ | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| New Zealand | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| Italy | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Scotland | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Armenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Moldova, Rep. of | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | $\cdots$ | - | - |  |  | 0 |
| Cyprus | - | - | - | $\bigcirc$ | $\bigcirc$ | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - $\cdot$ | - | - ${ }^{-}$ | - | - | - | - | - |  |  | 0 |
| Norway | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | $\cdots$ |  |
| Lebanon | - | - | - | - | - | - | - | - | - $\square^{-1}$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - ${ }^{\circ}$ | - | - | - | - | - | - | - |  |
| Macedonia, Rep. of | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - - | - | - | $\bigcirc$ | - | - | - | - |  |
| Jordan | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | $\stackrel{\square}{ }$ | - | - | - | - | ${ }^{\circ}$ | - | (-) | - | $\bigcirc$ | - | - | - | - | - | - | - |  |
| Indonesia | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - |  |
| Egypt | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - |  |
| Iran, Islamic Rep. of | - | - | $\cdots$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - - | - | - | - | - | - | - | - |  |
| Bahrain | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Tunisia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{\circ}$ | - | - $\cdot$ | - |  | - | - | - | - | - |  |
| Palestinian Nat'l Auth. | $\bigcirc$ | ${ }^{\circ}$ | - | - | - | - | - | - | - ${ }^{-}$ | - | - | - | - | ${ }^{\circ}$ | - | - | - | - | - ${ }^{-}$ | - | - ${ }^{-}$ | - | - | ${ }^{\circ}$ | - | - | ${ }^{\circ}$ | ${ }^{\circ}$ |  |
| Philippines | - | - | - | - | - | - | - | - | - | - | - | $\cdots$ | - | - | - | - | - | - | - ${ }^{-1}$ | - | - - | - | - | - | - | - | - | - |  |
| Morocco | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Chile | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | - | - | - | - | - |  |
| Botswana | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Saudi Arabia | - | $\bigcirc$ | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - |  |
| South Africa | - | - | - | $\bigcirc$ | $\bigcirc$ | - | - | - | - ${ }^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{\circ}$ | - | - | - | - | - | - | - |  |
| Ghana | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - |  |

Benchmarking Participants

| Basque Country, Spain | - | $\bigcirc$ | - | $\checkmark$ | - | $\bigcirc$ | $\checkmark$ | - | $\bigcirc$ | © | - | - | - | $\bigcirc$ | - |  |  |  |  |  | 0 | 0 |  |  | 0 | 0 | 0 | 0 | 0 | 0 |
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| Indiana State, US | (1) | (1) | - | ( | ( | - | - | - |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ontario Province, Can. | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | - | - | - | 0 | 0 | - | 0 | - | 0 | 0 | - | - | - | 0 |
| Quebec Province, Can. | ( | (1) | - | - | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

[^0]Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  | $\begin{aligned} & \text { ᄃ } \\ & \stackrel{\pi}{0} \\ & \text { on } \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{2} \\ & \underset{\sim}{3} \end{aligned}$ |  |  | $\frac{\frac{\pi}{n}}{\stackrel{y}{5}}$ |  | $\begin{aligned} & \text { y } \\ & \frac{1}{\bar{\circ}} \\ & \frac{0}{2} \\ & \frac{1}{a} \end{aligned}$ |  | $\frac{\stackrel{0}{\bar{J}}}{\frac{1}{\tau}}$ | 0 $\frac{0}{5}$ 3 3 0 0 |  |  |  |  | ~ <br>  |  |  | Countries |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Hungary |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | ( | Lithuania |
| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 |  |  | ( | United States |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | ( | Malaysia |
| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | (1) | ( | Israel |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | - | Serbia |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | - | - | - | England |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | - | (1) | ( ) | Sweden |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | (1) | (1) | - | Bulgaria |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | () | (1) | (-) | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | - | 0 |  | (1) | (1) | - | New Zealand |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ( | (1) | (1) | ( | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | ( | (1) | - | ( | Scotland |
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| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ( | (1) | (1) | ( | Moldova, Rep. of |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | - | (1) | (-) | Cyprus |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | (1) | (1) | (-) | Norway |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | (1) | - | Lebanon |
|  |  | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Macedonia, Rep. of |
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| © | - | - | - | © | (-) | - |  |  |  |  |  | 0 | 0 | 0 |  | - | - | - | - | Palestinian Nat'I Auth. |
| ( ${ }^{( }$ | - | - | - | (1) | - | - |  |  |  |  |  | 0 | 0 | 0 |  | - | (1) | - | - | Philippines |
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| - | (1) | (1) | (1) | (1) | ( | ( | ( ${ }^{( }$ | - | (1) | - | - | ( ${ }^{\text {c }}$ |  | 0 | 0 | ( | (1) | - | - | Saudi Arabia |
| ( ) | - | - | - | - | - | - | - | - | - | - | - | - | - |  | 0 | © | - | - | - | South Africa |
| (-) | (-) | - | (1) | (1) | - | (-) | - | - | - | - | - | (-) | (1) | - |  | - | - | (1) | ( ) | Ghana |

- Average achievement significantly higher than comparison country
(v) Average achievement significantly lower than comparison country

| - | 0 | - | - | - | 0 | - | - | - | - | 0 | - | - | - | 0 | 0 |  | (1) | ( ) | ( ) | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | (7) | Indiana State, US |
| 0 | - | - | - | 0 | 0 | - | 0 | - | - | 0 | - | - | - | - | - | 0 |  |  | (7) | Ontario Province, Can. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Quebec Province, Can. |

[^1]
no significant difference in performances; and a circle with a triangle pointing down indicates significantly lower performance.

At the eighth grade, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had significantly higher achievement in the knowing domain than the other participating countries. With the exception of those four top-performing countries, Japan had significantly higher achievement than all the rest of the participating countries. Estonia, Belgium (Flemish), and Hungary also performed very well as did the Canadian province of Quebec, being outperformed by only the five top-scoring Asian countries.

At the fourth grade, Singapore had the highest average achievement in the knowing domain followed by Hong Kong SAR and then Chinese Taipei and Japan. Belgium (Flemish) outperformed all the participating countries except the four top-scoring Asian countries. England, the Netherlands, the United States, and the US state of Indiana also had higher average achievement than many of the other participating countries.

## Applying Knowledge and Conceptual Understanding

Exhibit 2.3 presents the distribution of student mathematics achievement in the cognitive domain of applying at the eighth (first page) and fourth (second page) grades.

At the eighth grade, led by Singapore, 24 countries and the four benchmarking participants had achievement in the applying domain significantly higher than the international average. Romania, Bulgaria, Norway, and Serbia performed no differently than the international average and 18 countries performed significantly below this average. At the fourth grade, also led by Singapore, 14 countries and the US state of Indiana had achievement significantly higher than the international average, two countries (Italy and Australia) and the two Canadian provinces had achievement similar to the international average, and 9 countries had achievement below it.


* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2)

Did not satisfy guidelines for sample participation rates (see Exhibit C.2)

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.


* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).

Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash $(-)$ indicates comparable data are not available

Exhibit 2.4: \begin{tabular}{ll}

Multiple Comparisons of Average Mathematics Achievement for Applying \& | mathemarics |
| :--- |
| Cognitive Domain | <br>

Grade <br>
\hline
\end{tabular}

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

| Countries |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \frac{\pi}{\frac{\pi}{2}} \\ & \frac{5}{5} \\ & \frac{5}{8} \end{aligned}$ |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \stackrel{\pi}{5} \\ & \frac{0}{\omega} \\ & \frac{0}{\omega} \end{aligned}$ | 츤 |  |  | $\begin{aligned} & \stackrel{\pi}{\frac{\pi}{0}} \\ & \frac{0}{2} \\ & \frac{0}{3} \end{aligned}$ |  |  |  | $\begin{aligned} & \frac{n}{2} \\ & \frac{0}{3} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Singapore |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Hong Kong, SAR | - |  |  |  | 0 | 0 | 0 | - 0 | - | 0 | 0 | 0 | - | 0 | - | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | - | 0 | - | 0 | 0 |
| Korea, Rep. of | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Chinese Taipei | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Japan | - | - | - | - |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Netherlands | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | - | 0 | - | - | - | - | 0 | - | 0 | - | 0 | 0 |
| Belgium (Flemish) | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Estonia | - | - | - | - | - |  | - |  |  |  | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Hungary | - | - | - | - | - | - | - ${ }^{\circ}$ | - |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Malaysia | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 | - | - | - | 0 | 0 | - | - | 0 | - | 0 |
| Australia | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Scotland | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 |
| Sweden | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Latvia | - | - | - | $\cdot$ | $\cdot$ | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 |
| England | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Russian Federation | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | - | 0 | 0 | - | 0 | 0 | 0 |  |
| Slovak Republic | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| United States | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Lithuania | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| New Zealand | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | - | - | - | - | 0 |
| Israel | - | - | - | - | - |  | - | - | - | - | - | - |  | - |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Slovenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | - | 0 | - | 0 | 0 | 0 | 0 |
| Italy | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 |
| Armenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  | - | - | - | - | - | - |  |  |  |  | - | - | 0 | 0 |
| Romania | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  | 0 | 0 |
| Bulgaria | - | - | - | - | - |  | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - |  |  |  |  |  | 0 |  |
| Norway | - | - | - | - | - |  | - | - | - | - | - | - | - | - $\square^{-}$ | - | - |  | - | - | - | - | - | - | - | - |  |  |  |  | 0 |  |
| Serbia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  | - |
| Moldova, Rep. of | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\square^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |
| Cyprus | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |
| Macedonia, Rep. of | $\bigcirc$ | - | - | - | - |  | - | - | - | - | - | - | - | - $\cdot$ | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Lebanon | $\bullet$ | - | - | - | $\cdots$ |  | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | $\bullet$ | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - |
| Jordan | - | - | - | - | - |  | - | - | - | - | - | - | - | - $\square^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Tunisia | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Iran, Islamic Rep. of | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Indonesia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Egypt | - | - | - | - | - |  | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | $\bullet$ | - | - | - | - | - | - | - | - | - | - | - | $\bullet$ |
| Bahrain | - | - | - | - | - |  | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Chile | - | - | - | - | - |  | - | - | - | - | - | - | - | - ${ }^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Palestinian Nat'l Auth. | $\cdot$ | - | - | - | - | - | - | - | - | - | - | - | - | - | $\cdot$ | - | - | $\cdot$ | - | - | - | - | - | - | - | $\cdot$ | - | - | - | - | - |
| Morocco | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Philippines | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Botswana | - | - | - | - | - | - | - | - | - | - | - | $\stackrel{\square}{\circ}$ | $\bigcirc$ | - $\cdot$ | $\bullet$ | - | - | $\stackrel{\square}{\circ}$ | $\bullet$ | - | - | $\bullet$ | $\bigcirc$ | $\bullet$ | $\bullet$ | $\bullet$ | $\cdot$ | $\bigcirc$ | $\cdot$ | - | - |
| Saudi Arabia | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Ghana | - | - | - | - | - |  | - ${ }^{-}$ | - | - | - | - | - | - | - $\square^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| South Africa | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |

## Benchmarking Participants

| Basque Country, Spain | ( | ( | ( | (1) | (1) | - | - | © | (1) | - | - | © | ( | ( | ( | - | ( ) | - | - | ( | - | $\checkmark$ |  |  |  |  | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indiana State, US | ( | (-) | - | ( | (-) | ( | ( ${ }^{\text {c }}$ | ( ) | (1) |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ontario Province, Can. | - | (-) | - | (1) | (1) | (-) | (1) |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Quebec Province, Can. | ( | - | - | (1) | ( ) |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

[^2]$\left.\begin{array}{|lll}\hline \text { Exhibit 2.4: } & \begin{array}{l}\text { Multiple Comparisons of Average Mathematics Achievement for Applying } \\ \text { Cognitive Domain }\end{array} & \begin{array}{l}\text { mathemarics } \\ \text { Grade }\end{array} \\ 0\end{array}\right)$

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  |  | $\frac{\frac{\pi}{5}}{5}$ |  |  | $\stackrel{\stackrel{\rightharpoonup}{2}}{\underset{\sim}{3}}$ |  | $\frac{\stackrel{0}{\bar{J}}}{\frac{1}{\bar{J}}}$ |  |  |  |  |  |  |  |  | n <br>  |  |  | Countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Singapore |
| 0 | - | 0 | 0 | 0 | - | - | 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | 0 | 0 | 0 | Hong Kong, SAR |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Korea, Rep. of |
| 0 | - | 0 | 0 | 0 | - | - | 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | 0 | 0 | 0 | Chinese Taipei |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Japan |
| 0 | - | - | - | 0 | 0 | 0 | 0 | - | 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | 0 |  | Netherlands |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | Belgium (Flemish) |
| 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 |  | - | Estonia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | - | Hungary |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | (1) | Malaysia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Australia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | Scotland |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | Sweden |
| 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 |  | - | - | Latvia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | England |
| 0 | - | - | - | 0 | - | - | 0 | - | - | - | - | - | - | - | - | 0 |  | - | - | Russian Federation |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Slovak Republic |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | United States |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Lithuania |
| 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 |  | - | - | New Zealand |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |  | (1) | - | Israel |
| 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | ( | (1) | (1) | Slovenia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | - | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | - | Armenia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | - | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 |  | - | - | - | Bulgaria |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | © | - | (1) | - | Norway |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Serbia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | (-) | - | Moldova, Rep. of |
| 0 | $\bigcirc$ | - | 0 | 0 | - | - | - | 0 | - | 0 | 0 | 0 | 0 |  | 0 | - | - | - | - | Cyprus |
|  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | (1) | - | (-) | - | Macedonia, Rep. of |
|  |  |  |  | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 |  | 0 | (-) | - | - | - | Lebanon |
|  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | - | - | © | - | Jordan |
| (1) |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\checkmark$ | ( | (1) | (1) | Tunisia |
| (1) | - |  |  |  |  | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 |  | 0 | $\checkmark$ | - | () | - | Iran, Islamic Rep. of |
| (1) | ( | (1) | (1) |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\checkmark$ | - | (1) | - | Indonesia |
| (1) | (-) | - | - | © |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | - | (1) | - | Egypt |
| (1) | ( ) | - | (1) | (1) | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Bahrain |
| ( ${ }^{( }$ | - | - | - | © | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | © | - | (1) | - | Chile |
| ( ) | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | - | - | - | - | Palestinian Nat'l Auth. |
| ( ) | - | - | - | © | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | - | - | (1) | - | Morocco |
| - | - | - | - | - | - | - | (1) | (1) |  |  |  |  | $\bigcirc$ | 0 | 0 | - | - | - | - | Philippines |
| ( | ( | - | - | - | ( | - | - | ( ) | - | - |  |  | 0 | 0 | 0 | (1) | - | (1) | - | Botswana |
| ( | ( ${ }^{( }$ | - | (1) | (1) | (-) | - | (1) | (1) | (1) | - | - | (1) |  | 0 | 0 | (-) | ( ${ }^{\text {c }}$ | (1) | - | Saudi Arabia |
| ( ${ }^{( }$ | - | - | - | © | - | ( ) | - | - | - | ( ) | - | (1) | - |  | 0 | © | - | (-) | - | Ghana |
| (1) | - | - | - | - | - | - | - | - | ( | - | - | (1) | - | - |  | - | ( ${ }^{\text {c }}$ | (1) | - | South Africa |

- Average achievement significantly higher than comparison country
(v) Average achievement significantly lower than comparison country

| - | 0 | - | - | 0 | 0 | - | - | - | - | 0 | - | - | - | 0 | 0 |  | (7) | () | () | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (7) | (7) | Indiana State, US |
| 0 | - | - | - | 0 | 0 | - | 0 | - | 0 | 0 | - | - | 0 | - | 0 | 0 | - |  | (7) | Ontario Province, Can. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Quebec Province, Can. |

[^3]Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.


Benchmarking Participants

| Indiana State, US | $\bigcirc$ | - | - | - | ( | (1) | (1) | - | - | ) |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ontario Province, Can. | - | - | - | - | - | (1) | - | - | v | - | $\bigcirc$ | (-) | - |  |  |  |  |  | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - |  |
| Quebec Province, Can. | - | - | - | - | - | $\bigcirc$ | - | $\checkmark$ | - | ) | - | - | - |  |  |  |  | $\bigcirc$ | 0 | - | - | 0 | - | - | - | 0 | 0 | - |  |

- Average achievement significantly higher than comparison country
- Average achievement significantly lower than comparison country

Exhibit 2.4 shows for the eighth (first two pages) and fourth (third page) grades how a country's average mathematics achievement in the applying domain compares to achievement in the other participating countries.

At the eighth grade, Singapore had the highest achievement in the applying domain. Hong Kong SAR, the Republic of Korea, and Chinese Taipei all performed equally well, but not as well as Singapore. These countries were followed by Japan with performance below that of the four top-scoring countries, but with significantly higher achievement in this domain than all of the other participants at the eighth grade. The Netherlands and Belgium (Flemish) were outperformed by the five top-scoring Asian countries, but also did very well.

At the fourth grade, results for the four Asian countries in the applying domain were nearly the same as for the knowing domain. The Singaporean students had the highest average achievement in the applying domain, followed by the students in Hong Kong SAR, who (with the exception of Singapore) had significantly higher achievement than students in the other participating countries. Japan and Chinese Taipei performed similarly to each other and had the next highest achievement after Hong Kong SAR. Compared to the knowing domain, however, several more countries performed similarly to each other and were in the second highest achieving group of countries. Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands had significantly higher achievement in the applying domain than the rest of the participating countries and benchmarking entities.

## Reasoning

The first and second pages of Exhibit 2.5 show the distribution of student mathematics achievement in the cognitive domain of reasoning at the eighth and fourth grades, respectively. Exhibit 2.6 shows, for the eighth and fourth grades (first two pages and third page, respectively), how a


* Represents years of schooling counting from the first year of ISCED Level 1 .
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.

Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
\# Did not satisfy guidelines for sample participation rates (see Exhibit C.2).
National Desired Population does not cover all of International Desired Population (see Exhibit C.1).



* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.


## Kore

Singapore





Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  | $\frac{. \frac{\mathrm{C}}{0}}{\frac{\frac{0}{N}}{\frac{1}{0}}}$ |  | $$ | $$ |  | $\text { ułn } \forall \text { liten ue!!u! } \ddagger \text { Sojed }$ |  | $\begin{aligned} & \frac{0}{n} \\ & \frac{0}{\frac{1}{2}} \\ & \end{aligned}$ |  |  | 0 0 0 0 0 0 0 | $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{0} \\ & \frac{0}{4} \\ & \frac{0}{0} \\ & \frac{0}{0} \\ & n \end{aligned}$ |  |  |  | ~ \# $\stackrel{y}{5}$ <br>  |  |  | Countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | Singapore |
| 0 | 0 | - | - | 0 | - | - | - | - | - | - | - | - | 0 |  | 0 | 0 | - | - | 0 | Korea, Rep. of |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Chinese Taipei |
| - | - | - | - | - | - | - | - | - | - | - | - | - | 0 |  | - | 0 | - | - | - | Japan |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | Hong Kong, SAR |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 | - | - |  | Netherlands |
| 0 | 0 | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 |  |  | Belgium (Flemish) |
| - | - | - | - | 0 | - | - | 0 | - | - | - | - | - | - | - | - | 0 | - |  | (-) | Hungary |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 |  | (7) | Estonia |
| - | - | - | - | - | - | - | - | - | - | 0 | - | - | 0 |  | - | 0 |  | ( $\downarrow$ | (-) | Australia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |  | (-) | (-) | Scotland |
| - | - | - | - | - | - | - | 0 | - | - | - | - | - | - | - | - | 0 |  | - | - | New Zealand |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | - |  | 0 | 0 |  | ( ) | (1) | England |
| 0 | - | - | - | - | 0 | 0 | - | - | - | 0 | - | - | - |  | - | 0 |  | (7) | (1) | Sweden |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (7) | United States |
| 0 | 0 | - | - | - | - | - | - | - | - | 0 | 0 | 0 | - | - | - | 0 |  | (-) | (-) | Slovak Republic |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (v) | (-) | Malaysia |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  | (-) | (-) | Latvia |
| 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 |  |  | ( | (-) | Russian Federation |
| - | - | - | - | 0 | - | - | 0 | 0 | - | - | - | - | - | - | - |  |  | (1) | (-) | Slovenia |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | ( $)$ | (7) | (7) | Lithuania |
| - | - | - | - | - | - | - | - | - | - | 0 | - | - | 0 | - | - |  | (1) | ( $\downarrow$ | (-) | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | (-) | (-) | (-) | Israel |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | (1) | ( ) | (7) | (-) | Norway |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | - | 0 | 0 | (7) | ( ) | ( ) | (1) | Bulgaria |
| 0 | - | - | - | 0 | 0 | 0 | - | - | - | 0 | - | - | - | - | - | (1) | (1) | () | (1) | Armenia |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | ( 7 | (-) | (7) | Serbia |
| - | - | - | - | - | - | - | - | - | - | 0 | - | - | 0 | - | - | $\checkmark$ | ( ) | - | (-) | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (v) | (-) | (1) | (-) | Cyprus |
| - | 0 | - | - | 0 | - | - | 0 | - | - | - | 0 | - | - | - | 0 | ( ) | (1) | (-) | (-) | Moldova, Rep. of |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | (i) | ( - | (-) | Macedonia, Rep. of |
|  |  | 0 | 0 | 0 | 0 | - | - | 0 | - | - | - | - | - | - | - | (7) | ( ) | (-) | (-) | Jordan |
| (7) | (7) |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | (-) | (-) | (1) | Bahrain |
| (7) | (1) | (-) |  |  |  | - | - | 0 | - | 0 | 0 | 0 | - | - | - | ( ) | ( ) | (7) | (-) | Iran, Islamic Rep. of |
| (7) | (1) | (-) |  |  |  |  |  | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | (7) | (-) | (-) | (7) | Lebanon |
| (7) | (1) | (7) |  |  |  |  |  |  | - | 0 | - | - | - | - | - | (1) | ( ) | (7) | (-) | Chile |
| (7) | (1) | (1) | (1) |  |  |  |  |  |  | 0 | 0 | 0 | - | 0 | 0 | (7) | ( ) | (1) | (1) | Indonesia |
| (7) | (7) | (7) | ( ) |  |  |  |  |  |  | 0 | - | - | - | - | - | (7) | () | (7) | (-) | Palestinian Nat'l Auth. |
| (7) | (1) | (-) | (7) | (1) |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | (v) | ( $)$ | (7) | (7) | Egypt |
| (7) | (1) | (7) | (7) | (1) | ( ) |  |  |  |  |  | - | - | - |  | - | (7) | (7) | (7) | (7) | Tunisia |
| (-) | (1) | (-) | (7) | (-) | (-) | (1) | (1) |  |  |  | 0 | 0 | 0 | 0 | 0 | (v) | (-) | (1) | (-) | Morocco |
| (7) | (1) | (1) | (1) | (1) | - | (1) | (1) | (1) | (1) | ( ) |  |  |  |  | 0 | $\checkmark$ | (1) | ( ) | (-) | Philippines |
| (-) | - | ( | (-) | (-) | - | (-) | - | (v) | (1) | (\%) |  |  |  |  | 0 | (v) | - | - | (-) | Botswana |
| $\checkmark$ | () | ( ) | (-) | (-) | - | () | - | - | (-) | - |  |  |  |  | - | - | - | - | (-) | Saudi Arabia |
| (7) | (1) | ( ) | (1) | (7) | (1) | (1) | (1) | (1) | (1) | (7) | (7) | (1) | (7) |  | 0 | (7) | () | (1) | (1) | Ghana |
| (7) | (1) | (7) | (7) | (7) | (1) | (7) | (1) | (7) | (7) | (7) | (7) | (7) | (7) | (7) |  | (7) | (1) | (7) | (7) | South Africa |

Benchmarking Participants

| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | - | 0 | 0 | 0 |  |  | (1) | (1) | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |  |  | $\checkmark$ | - | Indiana State, US |
| 0 | - | - | - | - | 0 | - | - | - | - | - | - | 0 | - | - | - | - | 0 | 0 | 0 | 0 |  | $\checkmark$ | Ontario Province, Can. |
| 0 | 0 | - | 0 | 0 | 0 | - | - | - | - | 0 | 0 | 0 | - | - | - | - | 0 | 0 | 0 | 0 | - |  | Quebec Province, Can. |

- Average achievement significantly higher than comparison country
(7) Average achievement significantly lower than comparison country

[^4]| Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries. |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Countries |  |  |  |  |  | $\begin{aligned} & \text { 喜 } \\ & \frac{\pi}{0} \\ & \underset{\sim}{4} \end{aligned}$ | $\begin{aligned} & \frac{n}{0} \\ & \frac{c}{\pi} \\ & \frac{\pi}{0} \\ & \frac{5}{4} \\ & \frac{0}{2} \end{aligned}$ |  |  |  |  |  |  |  |  | $\frac{\pi}{\mathbb{\#}}$ |  |  |  |  |  | $\begin{aligned} & \stackrel{\rightharpoonup}{\circ} \\ & \dot{\circ} \\ & \stackrel{y}{c} \\ & \stackrel{u}{c} \\ & \frac{\pi}{v} \\ & \stackrel{\rightharpoonup}{c} \\ & \underline{0} \end{aligned}$ |  |  | $\frac{\frac{0}{n}}{\frac{1}{5}}$ |  |  |  |
| Singapore |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Hong Kong, SAR |  |  |  |  | 0 | 0 | 0 | - | - | 0 | 0 | 0 | 0 | - | - | 0 | 0 | - | 0 | - | - | - | - | - | - | 0 | 0 | - |
| Chinese Taipei |  |  |  |  | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Japan | ( ) |  |  |  | 0 | 0 | 0 | - | - | - | - | 0 | 0 | - | 0 | - | - | - | 0 | - | - | - | - | - | - | 0 | - | - |
| Belgium (Flemish) | (1) | ( | (1) | (\%) |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| England | ( ) | (1) | (1) | (1) |  |  |  |  | 0 |  | 0 | 0 | 0 | 0 | 0 | - | - | 0 | - | 0 | 0 | - | - | - | 0 |  | 0 | - |
| Netherlands | (1) | - | (1) | (1) |  |  |  |  | 0 |  | 0 | 0 | 0 | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |
| Latvia | (1) | - | (1) | (1) | (1) |  |  |  |  |  |  | - | - | - | 0 | - | - | - | - | 0 | - | - | - | - | - |  |  | - |
| Lithuania | (1) | () | (1) | (1) | (1) | (1) | ( ) |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 |
| Russian Federation | (1) | - | (1) | (-) | (1) |  |  |  |  |  |  |  |  | - | - | - | - | - | - | - | - | - | - | - | - |  |  | - |
| Hungary | (1) | (-) | (1) | (-) | (1) | (1) | (-) |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 |
| United States | ( $\downarrow$ | (1) | (1) | (-) | (1) | (1) | (-) | (1) |  |  |  |  |  | 0 | 0 | - | 0 | - | 0 | - | 0 | - | - | - | - | (1) |  | 0 |
| Cyprus | (1) | ( ) | (1) | (1) | (7) | (7) | (1) | (-) | (1) |  | $\nabla$ |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (7) |  |  |
| Australia | (1) | (-) | (1) | (1) | (1) | (1) | (1) | (1) | (1) | (1) | ( ) | (7) | (1) |  |  |  |  | - | 0 | - | - | - | - | - | - | (v) | (1) |  |
| New Zealand | (1) | (1) | (-) | (1) | (7) | (1) | (1) | (-) | (1) | (-) | (1) | ( $\uparrow$ | - |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (v) | (1) | (\%) |
| Italy | ( ) | ( | (-) | (-) | (-) | ( ) | (-) | (-) | ( ) | - |  | (v) | - |  |  |  |  |  | 0 | - | - | - | 0 | 0 | 0 | (v) | (1) | (-) |
| Scotland | (1) | (1) | (1) | (-) | (7) | (7) | (1) | (1) | (1) | (-) | (1) | ( $\downarrow$ | ( $)$ |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | (1) | ( ) |
| Moldova, Rep. of | (1) | (1) | (-) | (1) | (-) | (-) | (1) | (1) | (1) | (1) | (1) | ( $\downarrow$ | () | (1) |  |  |  |  |  | 0 | 0 | - | - | - | - | (1) | (1) | ( ) |
| Slovenia | ( ) | ( | (1) | (-) | (-) | (v) | (-) | - | - | (v) | () | - | ( - | (v) | $\bigcirc$ | (\%) | ( |  |  | 0 | 0 | 0 | 0 | 0 | 0 | (v) | ( | - |
| Norway | (1) | - | (1) | (1) | (1) | (-) | (1) | - | - | (1) | ( ) | (1) | (1) | (1) | (v) | - | ( ) | (1) | (1) |  | - | - | - | - | - | (1) | (7) | (1) |
| Armenia | (1) | ( ) | (1) | (1) | (1) | (1) | (1) | (1) | (1) | ( ) | (7) | (1) | ( $)$ | (1) | () | (1) | () | (1) | (1) | (1) |  | 0 | 0 | 0 | 0 | () | (1) | (1) |
| Iran, Islamic Rep. of | (1) | (7) | (-) | (1) | (1) | (1) | (1) | (1) | (1) | ( ) | - | (7) | (-) | (1) | () | ( | (1) | (7) | (1) | (1) | (1) |  | - | - | - | ( ) | (1) | (1) |
| Morocco | (1) | (1) | (1) | (-) | (7) | (7) | (-) | (1) | (1) | (-) | (1) | (-) | ( 7 | (1) | () | ( ) | (1) | (\%) | (1) | (1) | (1) | (-) |  |  | 0 | (v) | ( | (1) |
| Philippines | ( $\downarrow$ | ( ) | (1) | (1) | (1) | (1) | (1) | (1) | (1) | ( $)$ | ( ) | (v) | ( | (1) | (1) | - | (1) | ( ) | (1) | (1) | - | (1) |  |  | 0 | (7) | ( $)$ | (1) |
| Tunisia | (1) | (1) | (1) | (-) | (-) | (-) | (-) | (1) | (1) | ( ) | - | (-) | ( $)$ | ( ) | (\%) | (1) | - | ( ) | (1) | (1) | ( $\downarrow$ | - | (1) | (1) |  | (v) | (v) | (1) |
| Benchmarking Participants |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Indiana State, US | (1) | (1) | (1) | (1) | ( ) |  |  |  |  |  |  | 0 | - | - | 0 | 0 | - | - | 0 | - | - | - | - | - | 0 |  |  | 0 |
| Ontario Province, Can. | (1) | (1) | (1) | (1) | (7) | (1) | ( ) |  |  |  |  |  |  | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | 0 |
| Quebec Province, Can. | (1) | () | (1) | (1) | (1) | (1) | (1) | (1) | (1) | ( ) | - | - |  |  | 0 | 0 | - | - | 0 | - | - | - | - | - | - | (\%) |  |  |

- Average achievement significantly higher than comparison country
(7) Average achievement significantly lower than comparison country
country's average mathematics achievement in the reasoning domain compares to achievement in the other participating countries.

At the eighth grade, average achievement in the reasoning domain ranged from 583 in Singapore to 287 in South Africa. Twentyfour countries and the four benchmarking participants performed significantly above the international average, three countries (Bulgaria, Armenia, and Serbia) performed comparably to the international average, and 19 countries performed significantly below the average.

At the eighth grade, looking at both Exhibits 2.5 and 2.6, it can be seen that the rank ordering of significant differences in achievement is rather complicated for the reasoning domain. Singapore and the Republic of Korea had the highest average achievement in the reasoning domain, nearly identical ( 583 and 582), but Singapore had a larger standard error ( 3.5 to 1.7 ). Thus, the Republic of Korea had significantly higher achievement than every participating country except Singapore and Chinese Taipei whereas Singapore (with the larger standard error) had higher average achievement than every participating country except the Republic of Korea, Chinese Taipei, and Japan. Chinese Taipei and Japan had the same average score (576) followed by Hong Kong SAR. Chinese Taipei (also with a relatively larger standard error of 4.2) did not perform statistically differently than the other three Asian countries, whereas a difference was found between the Republic of Korea and Japan due to their small standard errors. Hong Kong SAR was outperformed only by Singapore and the Republic of Korea. The Netherlands and Belgium (Flemish) only were outperformed by the five top-scoring Asian countries.

At the fourth grade, performance ranged from 574 for Singapore to 340 for Tunisia. Fifteen countries and the three benchmarking participants performed significantly above the international average, three countries (Italy, Scotland, and Moldova) performed essentially at the international average, and seven countries performed significantly below the international average. Singapore had the highest achievement, outperforming all countries except Hong Kong SAR and

Chinese Taipei. Hong Kong, Chinese Taipei and Japan had similar achievement followed by Belgium (Flemish), England, and the Netherlands (all with similar average achievement and only outperformed by the four highest-achieving Asian countries).

## Overview Across Domains

At both the eighth and fourth grades, the countries with the highest achievement in each of the three cognitive domains also tended to be the highest-scoring countries (though not always in the same rank order) on the overall mathematics assessment. At the eighth grade (see Exhibit 1.1), the four countries with the highest overall mathematics achievement were Singapore followed by the Republic of Korea, Hong Kong SAR, and Chinese Taipei (only outperformed by Singapore). Japan had the next highest achievement outperforming all the rest of the participating countries except the previous four countries. Belgium (Flemish), the Netherlands, Estonia, Hungary, and the Canadian province of Quebec also performed well (at least as well or better than all other participants except the five Asian countries listed above).

- In knowing, similar to overall mathematics achievement, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement followed by Japan (see Exhibits 2.1 and 2.2). As a slight difference compared to the results for overall mathematics achievement, the four top-scoring Asian countries performed similarly to each other in the knowing domain. Estonia, Belgium (Flemish), Hungary, and the Canadian province of Quebec were outperformed only by the five top-achieving Asian countries.
- In applying, Singapore had the highest average achievement followed by Hong Kong SAR, the Republic of Korea, and Chinese Taipei and then Japan (see Exhibits 2.3 and 2.4). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec were next (only outperformed by the five top-achieving Asian countries).
- In reasoning, Singapore and the Republic of Korea performed very similarly followed by Chinese Taipei and Japan and then Hong Kong SAR (see Exhibits 2.5 and 2.6). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec also had relatively high achievement, only being outperformed by the five Asian countries.

At the fourth grade, Singapore was the highest-performing country in overall mathematics followed by Hong Kong SAR, and then by Japan and Chinese Taipei who performed similarly (see Exhibit 1.1). Belgium (Flemish) had higher achievement than all countries except these four Asian countries.

- In knowing, the pattern at the fourth grade was the same as for overall mathematics (see Exhibits 2.1 and 2.2). The four Asian countries had the best achievement (Singapore followed by Hong Kong SAR, and then by Chinese Taipei and Japan) with Belgium (Flemish) having higher achievement than all countries except the four bestachieving Asian countries.
- In applying, the pattern for the four high-achieving Asian countries was the same as for overall mathematics (see Exhibits 2.3 and 2.4). However, Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands all followed, performing similarly to each other with lower achievement than the four Asian countries, but higher achievement than the rest of the participating countries.
- In reasoning, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement (see Exhibits 2.5 and 2.6). Japan had achievement similar to Hong Kong SAR and Chinese Taipei, but was outperformed by Singapore. Belgium (Flemish), England, and the Netherlands had achievement equal to or higher than all participants except the four top-achieving Asian countries.

Just as countries with high achievement on the mathematics assessment as a whole had high achievement in the three cognitive domains, countries scoring lowest on the assessment as a whole
(e.g., South Africa, Ghana, and Saudi Arabia at the eighth grade and the Philippines, Morocco, and Tunisia at the fourth grade) also tended to have low performance in all three cognitive domains. For some middle-performing countries, however, performance was more varied. For example, at the eighth grade, Armenia performed above the international average in the knowing and applying domains (480 and 478) but essentially at the international average in the reasoning domain (468).

Looking at the range in scale scores across the cognitive domains at the eighth grade, the differences in average achievement between the highest- and lowest-performing countries were largest in the knowing domain ( 360 score points), next largest in the applying domain (342), and smallest in the reasoning domain (296). As described in the following sections, several more countries performed significantly above the international average in the knowing domain than in the applying and reasoning domains.

At the fourth grade, with fewer countries, the range in performance between the highest- and lowest-performing countries was smaller than at the eighth grade, but the pattern was similar. The largest difference was in the knowing domain (288), next in the applying domain (246), and the smallest difference was in the reasoning domain (234). In each of the three cognitive domains, about the same number of participants performed above, similar to, or below the international average.


[^0]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

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[^2]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^3]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

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